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# PROPERTIES OF CONIC SECTIONS PROVED GEOMETRICALLY.

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# PROPERTIES OF CONIC SECTIONS

## PROVED GEOMETRICALLY.

## PART I. THE ELLIPSE,

*WITH AN AMPLE COLLECTION OF PROBLEMS.*

BY

REV. HENRY GEORGE DAY, M.A.,

HEAD-MASTER OF SEDBERGH ROYAL FREE GRAMMAR SCHOOL;
AND FORMERLY FELLOW OF ST JOHN'S COLLEGE, CAMBRIDGE.

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# PREFACE.

My object, in presenting the following pages to the Public, is the introduction of a treatment of Conic Sections which should be simple and natural and lead by an easy transition to the analytical methods, without departing from the strict Geometry of Euclid.

I have therefore constantly added notes, to recall the Analytical formula, which the Student may read or omit, without interrupting the sequence of the Propositions.

Considerable advantage has been gained towards clearness of proof by establishing in the Introduction a few elementary Lemmas, which are constantly recurring in Geometrical investigations, but have not been proved independently by Euclid.

The work has been completed by a copious selection of problems, which will, it is hoped, prove interesting to the learner, and stimulate to the acquisition of Geometrical power.

H. G. DAY.

Sedbergh,
*March*, 1868.

# THE ELLIPSE.

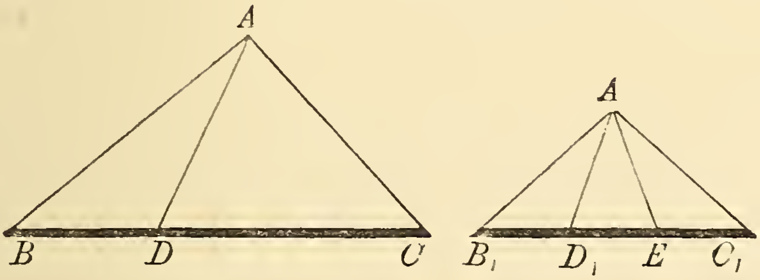
## INTRODUCTION.

### LEMMA I.

*The straight lines, which divide similarly the vertical angles of similar triangles, divide the bases proportionally.*

Let $ABC$, $A_{,}B_{,}C_{,}$ be two similar triangles, and let $AD$, $A_{,}D_{,}$ divide the vertical angles, $A$ and $A_{,}$, similarly;

then $$BD : DC :: B_{,}D_{,} : D_{,}C_{,}.$$



For, the triangles $ABD$, $A_{,}B_{,}D_{,}$ being manifestly similar,

$$AD : A_{,}D_{,} :: BD : B_{,}D_{,}.$$ [E. vi. 4.

Similarly, $$AD : A_{,}D_{,} :: DC : D_{,}C_{,};$$

$$\therefore BD : DC :: B_{,}D_{,} : D_{,}C_{,}.$$ Q. E. D.

Cor. 1. Conversely, if in the similar triangles $ABC$, $A_{,}B_{,}C_{,}$,

$$BD : DC :: B_{,}D_{,} : D_{,}C_{,};$$

then the angles, $BAC$, $B_{,}A_{,}C_{,}$, are similarly divided.

For, if not, make the angle $B_{,}A_{,}E = BAD$; therefore, by the previous part,

$$\begin{aligned} B_{,}E : EC_{,} &:: BD : DC \\ &:: B_{,}D_{,} : D_{,}C_{,}, \end{aligned}$$

which is impossible; therefore $AD$, $A_{,}D_{,}$ divide the angles similarly. Q. E. D.

Cor. 2. Any corresponding lines in the two triangles will be manifestly in the same ratio as

$$AB : A_{,}B_{,}.$$

Hence $AD^2 : A_{,}D_{,}^2 :: BD.DC : B_{,}D_{,}.D_{,}C_{,}$.

N.B. Other ratios of the same kind may readily be proved, and could be assumed if necessary.

## Lemma II.

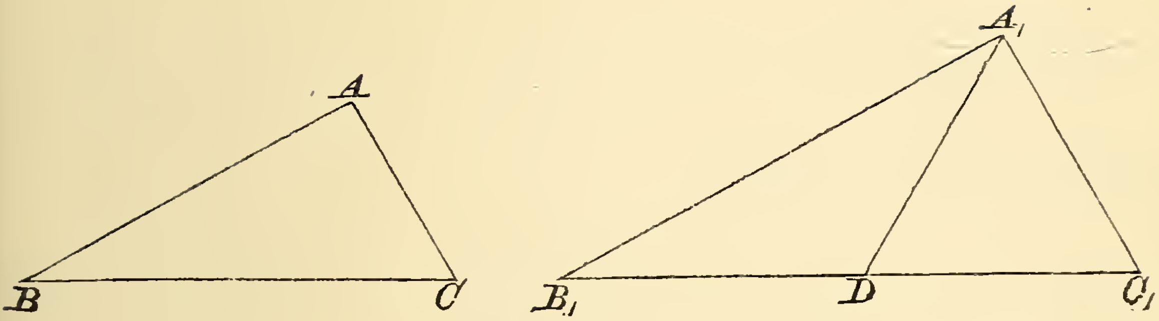
*If two triangles have an angle of the one equal to one angle of the other, and the sides about two other angles proportionals, the remaining angles shall be either equal or supplementary.*

1st. If they are both obtuse, or both acute, or if one of them is a right angle, they are equal. [E. vi. 7.

But, if $AB : AC :: A_{,}B_{,} : A_{,}D$,

and the angle $ABC = A_{,}B_{,}D$, where $D$ is an obtuse angle and $C$ acute; at the point $A_{,}$ make $\angle B_{,}A_{,}C_{,} = BAC$;

$$\therefore BA : AC :: B_{,}A_{,} : A_{,}C_{,}$$ [E. VI. 4.



$$:: B_{,}A_{,} : A_{,}D;$$

$$\therefore A_{,}C_{,} = A_{,}D.$$

Hence $A_{,}DC_{,}$ is an isosceles triangle;

$$\therefore \angle A_{,}DC_{,} = A_{,}C_{,}D = ACB; \text{ by 1st case.}$$

But $ADB_{,} + ADC_{,} = 2$ right angles;

$$\therefore ADB_{,} + ACB = 2 \text{ right angles.}$$ Q. E. D.

COR. 1. In the particular case where the sides

$$BA, AC = BA_{,}, A_{,}D,$$

and $\angle B = B_{,}$;

then the other angles $C$ and $D$ are either equal or supplementary.

COR. 2. If two right-angled triangles have the hypotenuses proportional to their altitudes, the base angles are equal.

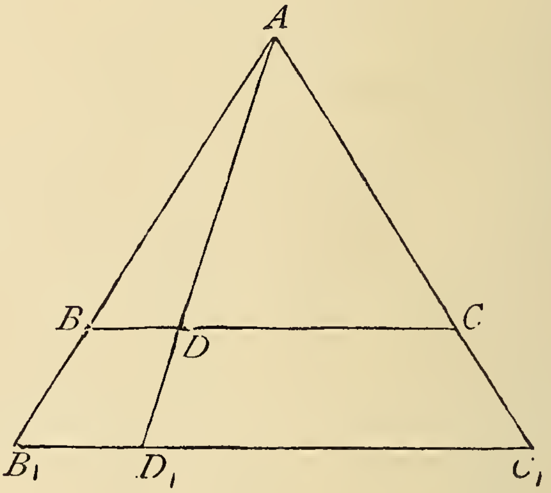
COR. 3. If two right-angled triangles have equal bases and hypotenuses, they have equal altitudes.

## LEMMA III.

*If two parallel straight lines are similarly divided, towards the same parts, the line joining the points of division and the lines joining their corresponding extremities will pass through one point.*

Let $BC$, $B_{,}C_{,}$ be two parallel lines similarly divided in $D$, $D_{,}$, so that

$$BD : DC :: B_{,}D_{,} : D_{,}C_{,} ;$$



and let $CC_{,}$, $BB_{,}$ meet in $A$; then $DD_{,}$ will pass through $A$.

For the triangles $ABC$, $AB_{,}C_{,}$, are manifestly similar, and their bases are similarly divided;

$$\therefore \angle BAD = B_{,}AD_{,} ;$$ [L. I. COR. I.

$$\therefore D_{,}, D, A, \text{ lie in one right line ;}$$

or $DD_{,}$ passes through $A$. Q. E. D.

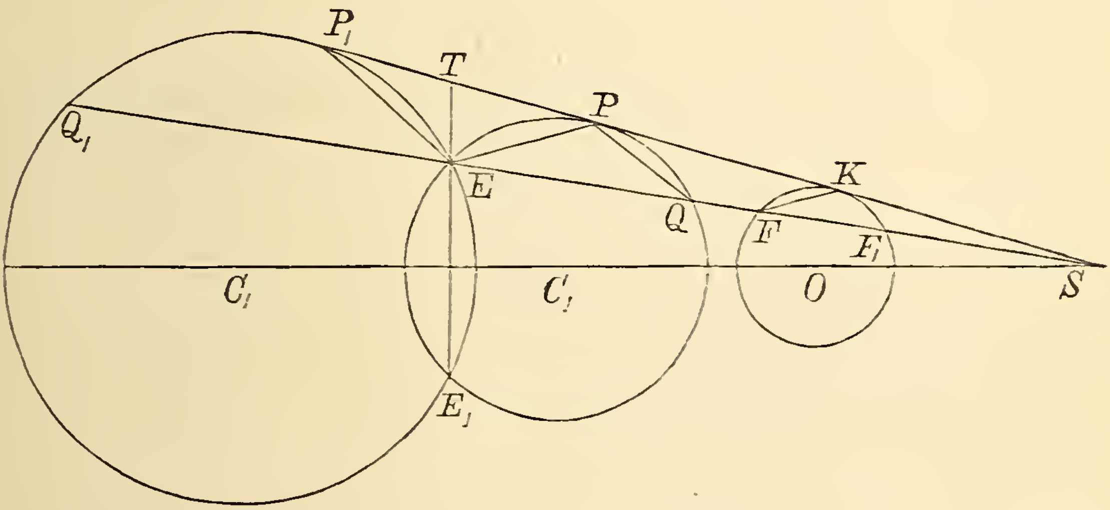
COR. 1. The same would hold, whatever the number of points of division.

COR. 2. If $BD = B_{,}D_{,}$ and $DC = D_{,}C_{,}$, then $BB_{,}$, $DD_{,}$, $CC_{,}$ are manifestly parallel.

## LEMMA IV.

*The common chord of two circles bisects the common tangent.*

Let $PTP_{,}$ be the common tangent to two circles, $TEE_{,}$ the common chord; $PT = P_{,}T$.



Because $PT$ touches the circle $PEE_{,}$;

$$\therefore\ PT^2 = TE\,.\,TE_{,}. \qquad \text{[E. III. 36.}$$

Similarly, $$P_{,}T^2 = TE\,.\,TE_{,}; \qquad \text{[E. III. 36.}$$

$$\therefore\ PT = P_{,}T. \qquad \text{Q. E. D.}$$

COR. If $EE_{,}$ be produced to meet in $T_{,}$ the other tangent through $S$ to the circle $EPQ$,

$$TP^2 = TE\,.\,ET_{,}.$$

For $$ET_{,} = E_{,}T;$$

$$\therefore\ TP^2 = TE\,.\,TE_{,} = TE\,.\,ET_{,}. \qquad \text{Q. E. D.}$$

## LEMMA V.

*The distance of either point of intersection of two circles from the centre of similitude, is a mean proportional between the tangents drawn to the circles from the centre of similitude.*

Let $S$ be the centre of similitude of the circles.

Then $SP : SE :: SE : SP_{,}$.

Let $SE$ meet the circle $PEE_{,}$ in $Q$.

Join $PQ$, $EP$, $EP_{,}$.

Then the $\angle SP_{,}E = SPQ$;

$= \angle$ in alternate segment $SEP$; [E. III. 32.

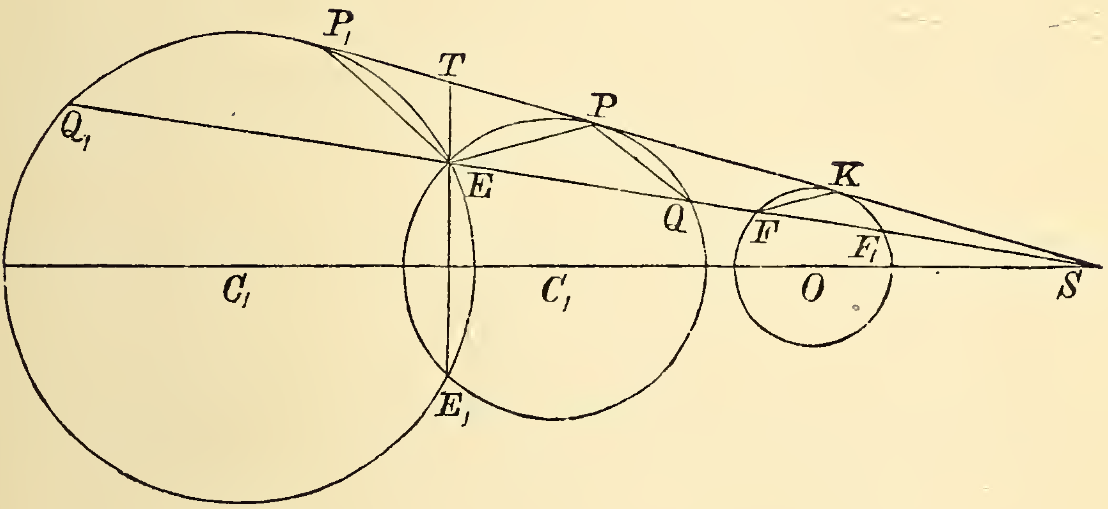
$\therefore$ the triangles $SPE_{,}$, $SEP_{,}$ are similar; and

$SP : SE :: SE : SP_{,}$. Q.E.D. [E. VI. 4.

## LEMMA VI.

*If three circles have a common centre of similitude, and, from the point of intersection of two of them, a secant be drawn to the third, the rectangle contained by the segments of that secant will be equal to the rectangle contained by the two common tangents drawn from the third circle to the two others.*

Let $FF_{\prime}K$ be another circle, with the same centre of similitude $S$.



Also let it touch $SP$ in $K$, and meet $SE$ in $F$, $F_{\prime}$;

$$EF \,.\, EF_{\prime} = PK \,.\, P_{\prime}K.$$

Join $FK$, which is parallel to $PE$;

$$\therefore\ EF : PK :: SE : SP. \qquad \text{[E. VI. 2.}$$

Similarly $\quad EF_{\prime} : P_{\prime}K :: SE : SP_{\prime}$;

$$\therefore\ EF \,.\, EF_{\prime} : PK \,.\, P_{\prime}K :: SE^2 : SP \,.\, SP_{\prime};$$

$$\therefore\ EF\ EF_{\prime} = PK \,.\, P_{\prime}K. \qquad \text{[L. 5.}$$

Similarly if any other secant be drawn through $E$ to the circle $FF_{\prime}K$, the rectangle under its segments will be equal to $EF \,.\, EF_{\prime}$; [E. III. 36.

or to $PK \,.\, P_{\prime}K$.

∴ If three circles &c. Q. E. D.

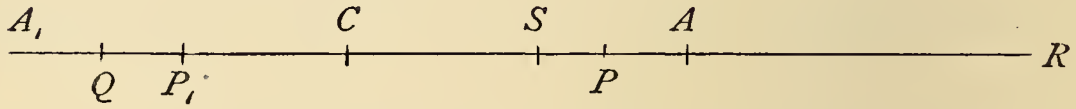
COR. I. Exactly the same proof would hold if the point $E$ were within the circle $KFF_{\prime}$, and $FE$, $EF_{\prime}$ were the rectangle under the segments of a chord through $E$.

COR. 2. Hence also if $O$, $C$, $C_{,}$ be the centres of the three circles,

$$EF, EF_{,} : OC \,.\, OC_{,} :: SK^2 : SO^2.$$

DEF. If, in the line $SR$, a point $A$ be taken so that

$$SA : AR :: e : 1;$$



$A$ is said to be the point in which $SR$ is divided internally in the ratio of $e : 1$.

Again, if $RS$ be produced to $A_{,}$ so that

$$SA_{,} : A_{,}R :: e : 1,$$

$A_{,}$ is said to be the point of external division of $SR$ in that ratio.

It is evident that $A_{,}$ lies on the side next $S$, and $SA_{,}$ is less than $A_{,}R$, if $e$ is less than 1.

In this investigation $e$ is supposed less than 1.

## LEMMA VII.

*If $AA_{,}$ are the points in which $SR$ is divided internally and externally in the ratio $e : 1$; then if $P$ be any point within $AA_{,}$;*

$$SP : PR \text{ is less than } e : 1.$$

1st. Let $P$ lie between $S$ and $A$.

Then $PS$ is less than $PA$, and $PR$ greater than $AR$; therefore for both reasons

$$SP : PR \text{ is less than } SA : AR;$$

$$\text{or less than } e : 1.$$

Next, let $P_{,}$ be a point between $S$ and $A$.

Take a point $Q$, so that

$$A_{,}Q : A_{,}P_{,} :: e : 1;$$

then $$SA_{,} : A_{,}R :: e : 1,$$ [Hyp.

$$A_{,}Q : A_{,}P_{,} :: e : 1;$$ [Const$^{n}$.

$$\therefore SQ : RP_{,} :: e : 1;$$

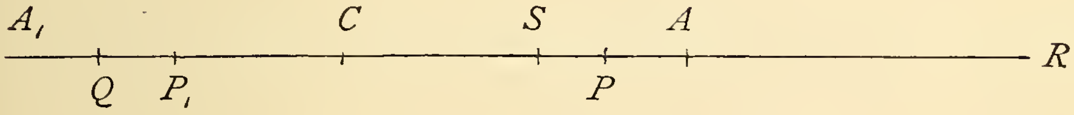
$$\therefore SP_{,} : RP_{,} \angle e : 1.$$ Q. E. D.

COR. 1. In exactly the same manner it may be proved that, if a point lies without $AA_{,}$, the ratio of these distances is greater than $e : 1$.

COR. 2. It is evident from this that two and only two points can be found dividing a line internally and externally in a given ratio.

### LEMMA VIII.

*If C is the middle point of* $AA_{,}$;



$$CS : CA :: e : 1,$$

*and* $$CA : CR :: e : 1.$$

For, $$SA : AR :: e : 1,$$

$$SA_{,} : A_{,}R :: e : 1;$$

$$\therefore SA + SA_{,} : AR + A_{,}R :: e : 1,$$

$$\begin{array}{lcl} AA_{,} : 2CR & :: & e : 1, \\ CA : CR & :: & e : 1. \end{array}$$

Again, $SA_{,} - SA : A_{,}R - AR :: e : 1,$

$$\begin{array}{lcl} 2CS : AA_{,} & :: & e : 1, \\ CS : CA & :: & e : 1. \end{array}$$

COR. 1. $CS \,.\, CR = CA^2.$

COR. 2. $AS \,.\, SA_{,} = CS \,.\, SR.$

For, $AS \,.\, SA_{,} = CA^2 - CS^2$ [E. II. 5.

$$= CS \,.\, CR - CS^2 \text{ [Cor. 1.}$$

$$= CS \,.\, SR.$$

COR. 3. $AS \,.\, SA_{,} : CA \,.\, SR :: CS : CA$

$$:: e : 1$$

$$:: SA : AR.$$

COR. 4. $SA_{,}\, SA_{,} : CA \,.\, SR :: e : 1.$

For $SA_{,} .\, SA_{,} = CS \,.\, SR;$ [COR. 2.

$$\therefore SA \,.\, SA_{,} : CA \,.\, SR :: CS \,.\, SR : CA \,.\, SR$$

$$:: CS : CA$$

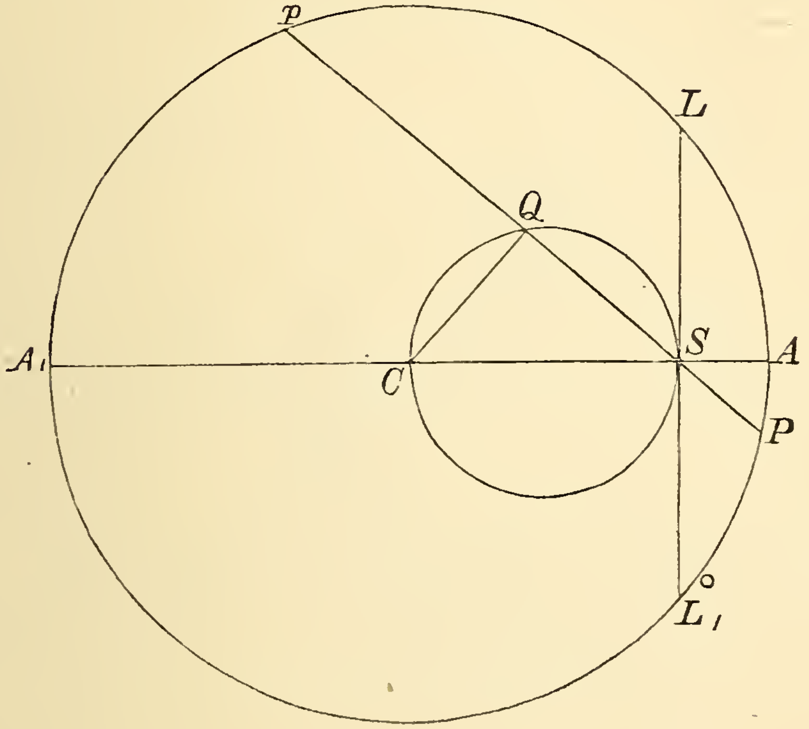
$$:: e : 1.$$

Q. E. D.

## LEMMA IX.

*The longest chord of a circle that passes through a given point is the diameter, and the shortest that perpendicular to the diameter; and, of the rest, that which is nearer to the diameter is greater than the more remote.*

Let $S$ be a point within a circle; $C$ the centre, $LL_{,}$ a chord perpendicular to $CS$.



Upon $CS$ as diameter describe a circle $CQS$, meeting a chord $Pp$ in $Q$. Join $CQ$.

Then because $CQS$ is a semicircle, the angle is a right angle; or $CQ$ is the distance from $C$ of the chord $Pp$.
[E. III. 31.

But, in the circle $CQS$, $CS$ is the longest chord through $S$,
[E. III. 15.

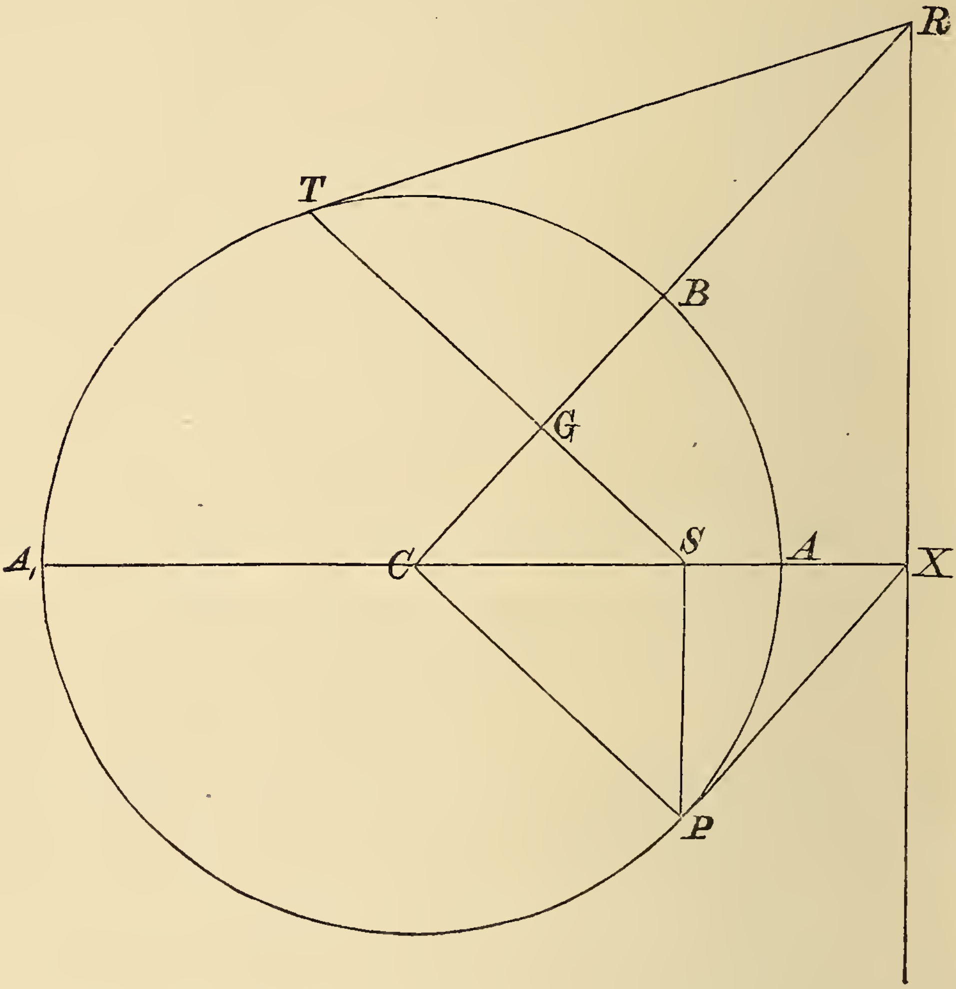
and, of the rest, that which is nearer $CS$ is longer than the more remote;
[E. III. 7.

But in the circle $pLA$ the shorter chord is further from the centre.
[E. III. 15.

Hence $LL_{,}$, whose distance is $SC$, will be the shortest chord of the circle $pLA$; and of the rest that chord which is nearer to $AA_{,}$ will be greater than the more remote. Q. E. D.

DEF. If two points $SX$ be taken in the diameter $AA'$ of a circle, so that

$$CS . CX = CA^2,$$



these points are called reciprocal points; and the lines $PS . RX$, perpendicular to the diameter, are called polars of $X$, $S$, respectively.

Also $X$, $S$, are called the poles of these lines.

## LEMMA X.

*The polar of any point, external to a circle, passes through the points of contact of tangents drawn to the circle from that point.*

Let $X$ be an external point, and let the polar of $X$ meet the circle in $P$. Join $SP$.

Then $CS \,.\, CX = CA^2 = CP^2$;

$\therefore\ CX : CP :: CP : CS$;

$\therefore$ the triangles $CPX$, $CSP$ are similar; [E. VI. 6.

$\therefore\ CPX$ is a right angle;

$\therefore\ PX$ is the tangent at $P$. [E. III. 16.

Similarly, the tangent at the other point in which $PS$ cuts the curve passes through $X$;

$\therefore$ the polar of an external point &c. Q. E. D.

COR. 1. Hence the line joining the points of contact of tangents from an external point is polar of that point.

COR. 2. Again, the line passing through one point of contact, and perpendicular to $CA$, will be polar of $X$.

## LEMMA XI.

*The polar of any point R, on the polar of S, passes through S.*

Join $RC$ and draw $SG$ perpendicular to $CR$.

Then by similar triangles

$CS : CG :: CR : CX$;

$\therefore\ CG \,.\, CR = CS \,.\, CX = CA^2 = CB^2$;

$\therefore\ R$ and $G$ are reciprocal points;

therefore $SG$, through $G$, perpendicular to $CB$, is the polar of $R$. But $SG$ passes through $S$.

Therefore the polar &c. Q. E. D.

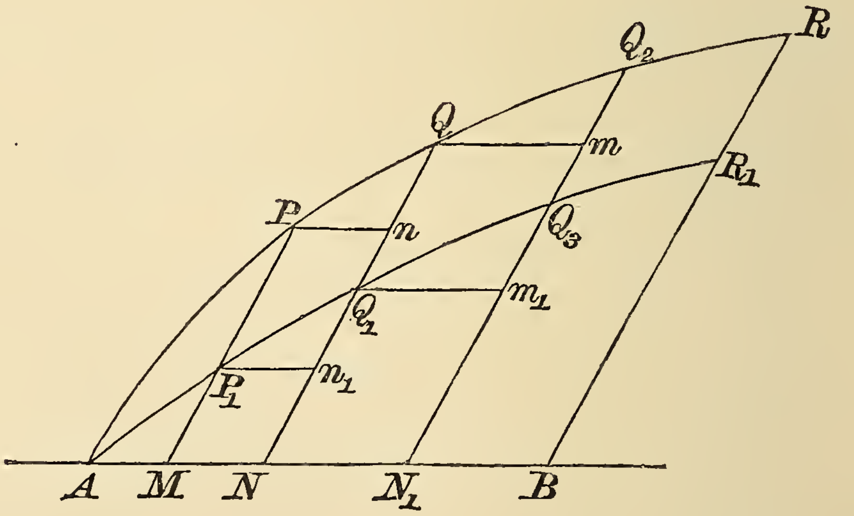
Cor. 3. Hence the line through $S$, perpendicular to $RC$, passes through the points of contact of tangents from $R$.

Def. If, through a point $P$ on a curve, a line $PQ$ be drawn meeting the curve in a point $Q$, near $P$, then, if $Q$ be taken nearer and nearer to $P$ continuously, the limiting position of $PQ$, when $Q$ moves up to coincide with $P$, is called the tangent at $P$.

## Lemma XII.

*If two curves on the same axis have the ordinates corresponding to the same abscissæ always in a constant ratio, the area of the curves will be in that ratio; and the tangents to the two curves at the extremities of the same ordinate will meet on the axis.*

Let $APQR$, $AP_{,}Q_{,}R_{,}$ be on the same axis $AB$, and let



$PP_{,}M$, $QQ_{,}N$ be ordinates parallel to $RR_{,}B$. Then if

$$PM : P_{,}M :: RB : R_{,}B$$

for each ordinate, the two areas will be in the ratio

$$RB : R_{,}B.$$

Also tangents at $P$, $P_{,}$, will meet in $BA$.

From $P$, $P_{,}$, draw $Pn$, $P_{,}n_{,}$ parallel to $AB$ to meet $QN$ in $n$, $n_{,}$.

Then the parallelogram

$$PN : P_{,}N :: RB : R_{,}B.$$

In the same manner, if $QN_{,}$ be another ordinate

$$QN_{,} : Q_{,}N_{,} :: RB : R_{,}B.$$

Now, if the number of these parallelograms be increased without limit, and the base of each diminished without limit, then the area of one set of parallelograms $PN$, $QN$, &c. will become equal to the area of the curve $APQRB$.

Similarly, the area of the other set will become equal to $AP_{,}Q_{,}R_{,}B$;

$$\therefore APQRB : AP_{,}Q_{,}R_{,}B :: RB : R_{,}B.$$

Also since $PM$, $QN$ are similarly divided in $P_{,}$, $Q_{,}$; $PQ$, $P_{,}Q_{,}$ will meet in $BA$. [L. 3.

This will be true if $N$ move up to $N_{,}$, and $Q$ consequently move up to coincide with $P$.

But then $PQ$, $P_{,}Q_{,}$ become ultimately tangents at $P$, $P_{,}$.

Hence the tangents at $P$, $P_{,}$ intersect in $BA$. Q. E. D.

COR. As a particular case, if $BA$ bisects all chords of a curve parallel to $RB_{\prime}$, then the tangents at the extremities of such a chord intersect on $BA$.

DEF. If, from a series of points on a plane, perpendiculars be drawn to another plane, inclined to the former at an oblique angle, the points which are the feet of these perpendiculars are called the projections of the former points.

DEF. The former points are said to be projected on this plane.

DEF. The lines joining each point with its projection are called the projecting lines.

DEF. The line formed by the projections of all the points of a given line (straight or curved) is called the projection of the given line.

DEF. The area of a projected figure is called the projection of the area of the figure from which it is projected.

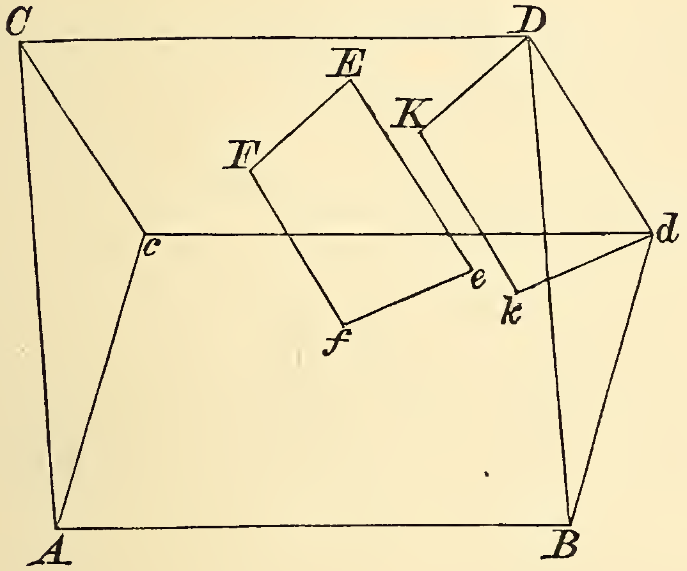
## LEMMA XIII.

*The projections of parallel straight lines are parallel straight lines, and their lengths are in the same ratio as those from which they have been projected.*

Let $EF$, a straight line on the plane $ABCD$, be projected into the line $ef$ on the plane $ABcd$.

Then the projecting lines of each point on $EF$ will be perpendicular to the plane $ABCD$, and therefore parallel to one another. [E. XI. 6.

Hence they will all lie in one plane *EFef* perpendicular to *ABcd*.



But the projection of *EF* will be determined by the intersection of this plane with the plane *ABcd*.

Hence all the points on *ef* will lie in one right line.
[E. XI. 3.

Or the projection of a straight line will be a straight line.

Next, let *DK*, any other straight line, parallel to *EF*, be projected into *dk*.

Then the plane *DKdk* is manifestly parallel to *EFef*,

since *Dd*, *Dk* are parallel to *EF*, *ef*;

∴ *Kd* is parallel to *ef*.

Also *KD*, *kd* being parallel to *EF*, *fe*, the ∠ between *KD*, *kd* will be equal to that between *EF*, *fe*; [E. XI. 10. and *Ddk* is a right angle, therefore = *Eef*.

Hence *DK* : *kd* :: *EF* : *ef*.

COR. I. Any straight line is divided by the projections of its points of division, similarly to that from which it is projected.

COR. 2. It is also manifest that straight lines parallel to $AB$ will be of the same length as their projection.

For if $cd$ be the projection of $CD$ parallel to $AB$, $cd$ will also be parallel to $AB$ or $CD$;

$$\therefore\ CDcd \text{ is a parallelogram, and } CD = cd.$$

COR. 3. Also, lines perpendicular to $AB$, the intersection of the planes will be projected into lines perpendicular to $AB$.

For let $BD$ be perpendicular to $AB$; $Bd$ its projection.

Then $Bd$, $Dd$ are each perpendicular to $Fd$; therefore $Fd$ is perpendicular to the plane $DdB$;

$\therefore$ $Bd$ is perpendicular to $AB$. Q. E. D.

## LEMMA XIV.

*The projection of a tangent to a curve is a tangent to the projected curve.*

For, a secant and its points of intersection will evidently be projected into a secant to the projected curve and its points of intersection.

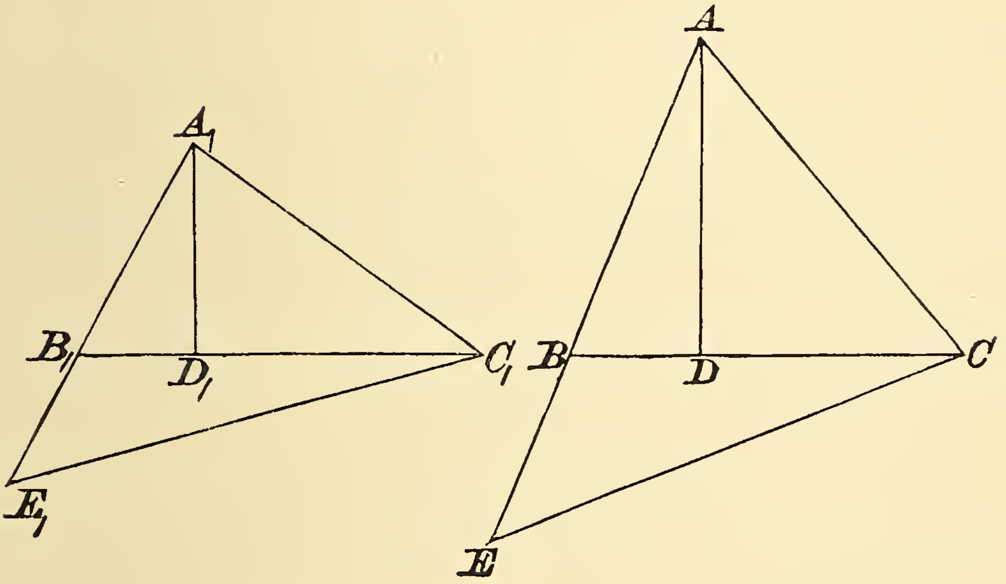
Now let the points of intersection move up to coincidence, and the secant become a tangent. Similarly, the points of intersection on the projected curve will move up to coincide.

Therefore the projected line will become a tangent to the projected curve. Q. E. D.

## LEMMA XV.

*Areas are projected proportionally.*

First let the areas be triangles $ABC$ and $A_{\prime}B_{\prime}C_{\prime}$ its projection; $BC$ being parallel to the intersection of the two planes.



Draw $AD$ perpendicular to $BC$, and let $A_{\prime}D_{\prime}$ be the projection of $AD$.

Then, because $BC$ is parallel to the intersection of the planes, its projection $B_{\prime}C_{\prime}$ will also be parallel to the intersection of the planes, and equal to $BC$. [L. 13. COR. 2.

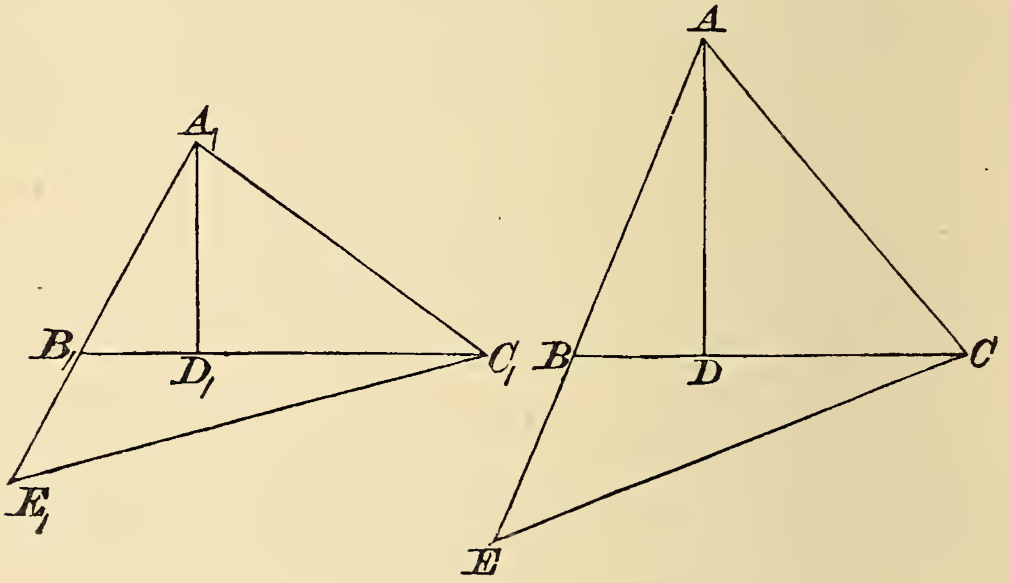
Again, since $A_{\prime}D_{\prime}$ is the projection of $AD$, $A_{\prime}D_{\prime}$ will be perpendicular to the intersection of the planes, and therefore to $B_{\prime}C_{\prime}$. [L. 13. COR. 3.

Also if $\alpha$ is the angle between the planes,

$$AD : A_{\prime}D_{\prime} :: 1 : \cos \alpha.$$

Hence the triangle $ABC : A_{,}B_{,}C_{,} :: 1 : \cos\alpha$. [E. VI. I.

Next, let the projected area be a triangle $ACE$, none of whose sides are parallel to the intersection of the planes.



Draw $CB$ parallel to the intersection.

Then the triangle

$$ABC : A_{,}B_{,}C_{,} :: 1 : \cos\alpha,$$

$$BCE : B_{,}C_{,}E_{,} :: 1 : \cos\alpha;$$

$$\therefore \Delta AEC : \Delta A_{,}E_{,}C_{,} :: 1 : \cos\alpha.$$

Similarly, any polygon and its projection are divisible into triangles of that ratio.

Therefore the two polygons will be in that ratio.

But the areas of curves are the limits of the areas of polygons of an indefinite number of sides.

Therefore the area of a curve and its projection are in the same ratio.

Therefore areas, &c. Q. E. D.

DEF. At any point of a plane curve, a circle that is drawn so as to coincide most nearly with the curve at that point is called the circle of curvature at that point.

DEF. The radius of this circle is called the radius of curvature at the point.

N.B. The radius of curvature and position of the circle of curvature vary for different points of the curve; each radius measuring the degree of convexity (or curvature) of the curve at a particular point.

DEF. A chord of the circle of curvature drawn in any direction is called the chord of curvature in that direction.

## LEMMA XVI.

*The circle of curvature at a point is the limiting position of the circle which passes through that point and two neighbouring points of the curve, when those two points each move up to the first mentioned point.*

For, it is evident that, by making the circle pass through as many consecutive points as possible, we get the circle to coincide most nearly with the curve. But three points will be sufficient to determine the circle.

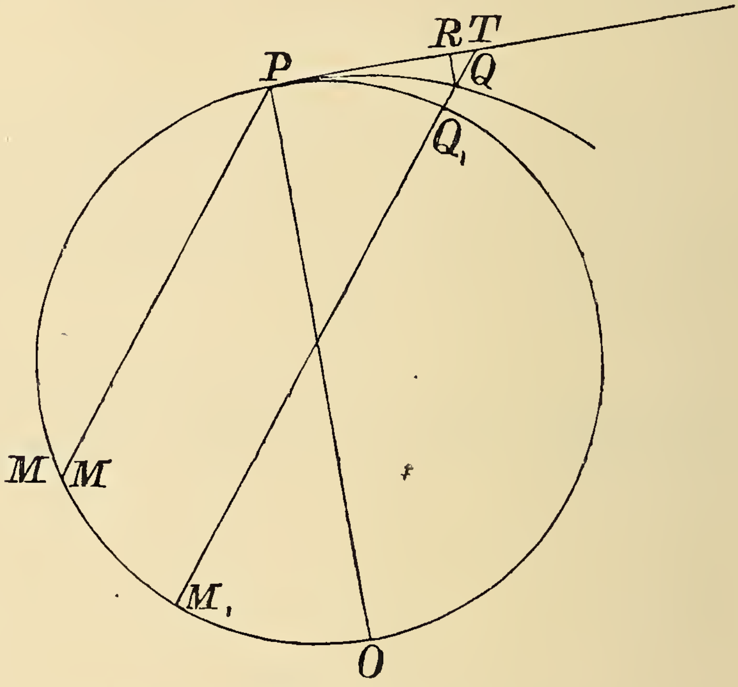
Therefore the circle of curvature, &c. Q. E. D.

COR. Hence the circle touches the curve at the point.

For they have the same tangent, viz. the line passing through two consecutive points, which will be common to the curve and to its circle of curvature.

## LEMMA XVII.

*If $PT$ be a tangent to a curve, then if from $T$ a line be drawn, making a finite angle with $PT$, and meeting the curve in $Q$, and if $PM$ be the chord of curvature parallel to $TQ$, then in the limit when $T$ moves up to $P$,*



$$TQ \, . \, PM = TP^2.$$

Let $TQ$ meet the circle in $Q_{,}M_{,}$.

Then, because the circle is to coincide as closely as possible with $PQ_{,}$ about the point $P$, the ratio $TQ : TQ_{,}$ will be ultimately unity when $T$ moves up to $P$.

Now, since the circle and curve have a common tangent at $P$, $PT$ will touch the circle;

$$\therefore \; PT^2 = TQ \, . \, TM,$$

$$= TQ_{,} \, . \, TM_{,}, \text{ ultimately.}$$

But $TM_{,}$ is ultimately $= PM$;

$$\therefore \; PT^2 = TQ \, . \, PM, \text{ ultimately.} \qquad \text{Q. E. D.}$$

N.B. This is generally expressed by saying that

$$PM = \text{limit of } \frac{PT^2}{TQ} \text{ when } PT = 0.$$

Cor. 1. The diameter of curvature being $PO$, and $QR$ being perpendicular to $PR$,

$$QR \cdot PO = PR^2.$$

For the diameter is the chord perpendicular to the tangent $PQ$.

Cor. 2. Chords of curvature equally inclined to the tangent are equal.

For chords of any circle, equally inclined to a tangent, and passing through the point of contact, are equal.

N.B. If $PO$ be the diameter of curvature, $PM$ any chord inclined at $\angle \theta$ to the tangent,

$$\theta = POM,$$

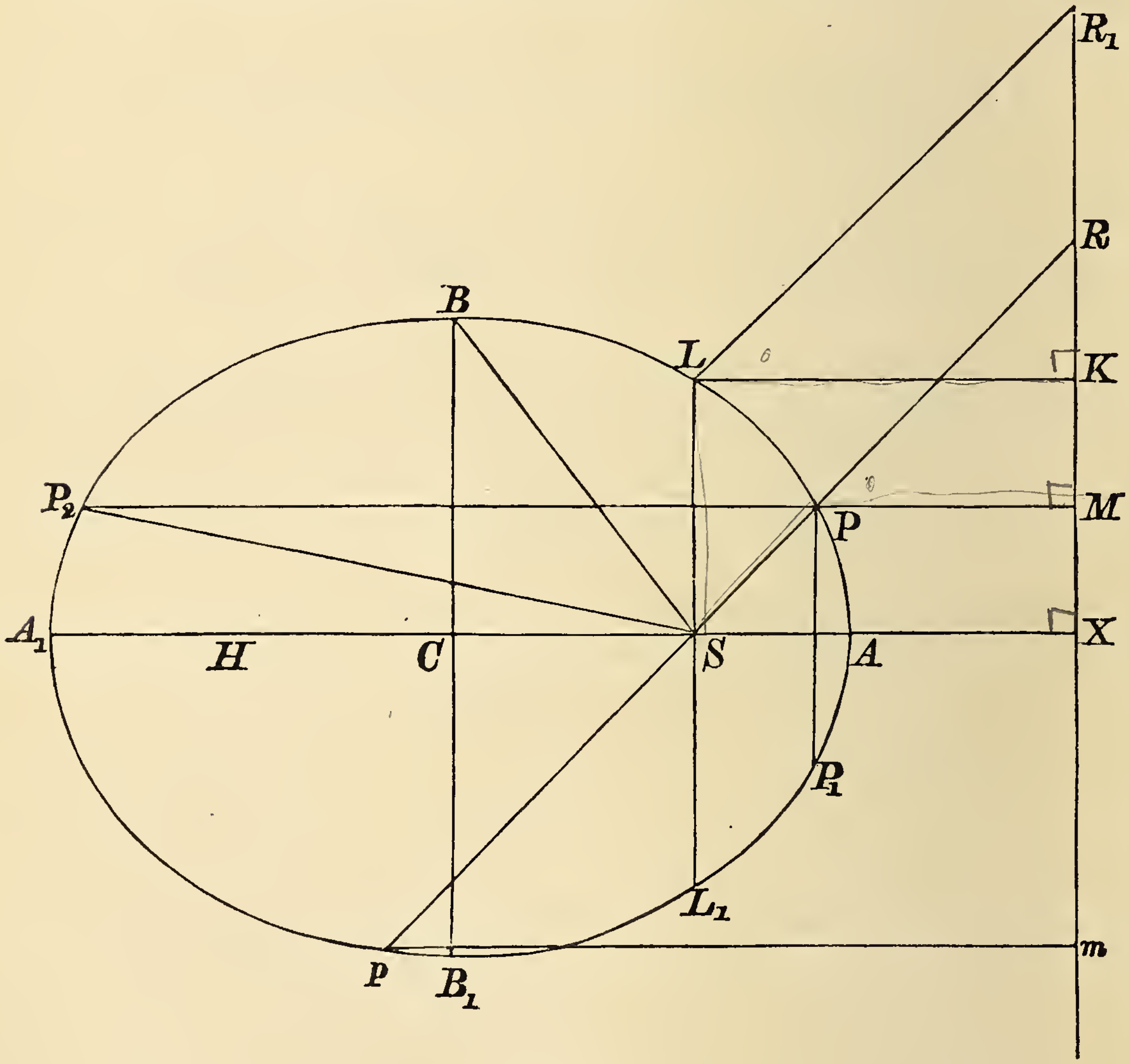
and $PMO$ is a right angle;

$$\therefore PM = OP \sin\theta.$$

## DEFINITIONS.

1. An *Ellipse* is the locus of a point, on a plane which contains a fixed point and a fixed right line, whose distance from the fixed point bears to its distance from the fixed line a constant ratio less than unity.

2. The fixed point is called the *Focus.*



3. The fixed right line is called the *Directrix.*

4. The constant ratio is called the *Excentricity*, and is denoted by the letter *e*.

N.B. If $APB$ be an ellipse, $XM$ its directrix, $S$ its focus, then, if any line $P_2PM$ be drawn perpendicular to $XM$,

$$SP : PM :: e : 1,$$

and $$SP_2 : P_2M :: e : 1.$$

5. The line joining a point to the focus is called the *Focal Distance* of the point.

6. A chord through the focus is called a *Focal Chord.*

## PROP. I.

*Parallel straight lines, drawn from points in the curve, to meet the directrix, are proportional to the focal distances of the points.*

Let $LP$ be two points on the curve; parallel lines, drawn from $L$ and $P$ to the directrix, will be to each other in the ratio of $SL : SP$.

First, let the lines be $PM$, $LK$, each perpendicular to the directrix.

Then $$SL : LK :: e : 1, \qquad [D. 1.$$

and $$SP : PM :: e : 1; \qquad [D. 1.$$

$$\therefore SL : LK :: SP : PM,$$

and $$LK : PM :: SL : SP.$$

Next let $PR$, $LR_1$ be any other two parallels.

Then the triangles $RPM$, $R_{,}LK$ are equiangular and similar;

$$\therefore LR_{,} : PR :: LK : PM$$ [E. VI. 4.

$$:: SL : SP.$$

Hence parallel, &c. Q.E.D.

COR. 1. If $pSPR$ be any chord through $S$,

$$SP : PR :: Sp : pR.$$

Also $SR$ is divided by the points $P$, $p$, internally and externally in the same ratio of lesser inequality.

COR. 2. By the same reasoning it is evident that if from any two points $QQ_{,}$, wherever situated, $QM$, $QM_{,}$ be drawn perpendicular to the directrix, and $QR$, $QR_{,}$ inclined at any other angle,

$$SQ : SQ_{,} \gtreqless QR : Q_{,}R_{,}$$

according as $$SQ : QM \gtreqless SQ_{,} : Q_{,}M_{,}.$$

## PROP. II.

*An ellipse is a closed curve, lying entirely on the same side of the directrix as the focus, and including the focus within it.*

For, from any point $R$ of the directrix, draw a line $RPSp$ through the focus.

Thus $P$, $p$ are the points in which the line $SR$ is divided internally and externally in a ratio of less inequality. [I. COR. I.

Hence there are two such points, and only two ; [L. 7. COR. 2.

and one lies between $S$ and $R$, and the other on $RS$ produced beyond $PS$.

Therefore the figure is a closed curve, &c. Q.E.D.

N.B. The shape of the ellipse will obviously be as represented in the figure.

## DEFINITIONS.

7. The straight line $XA$, $SA_{\prime}$, drawn through the focus, perpendicular to the directrix, is called the *Major Axis* of the curve.

8. The point $C$, bisecting $AA_{\prime}$, is called the *Centre.*

9. Any chord through the centre is called a *Diameter.*

10. The points, where a diameter meets the curve, are called the *Vertices* of the diameter.

11. The part of a diameter, intercepted between its vertices, is called its *Length.*

12. The diameter $BB_{\prime}$, perpendicular to the major axis, is called the *Minor Axis.*

13. The point, where a chord meets the directrix, is called the *Foot* of the chord.

14. A line drawn from a point on the curve, perpendicular to either axis, is called an *Ordinate* to that axis.

15. The parts, into which an axis is divided by the ordinate passing through a point on the curve, are called the *Abscissæ* of that point.

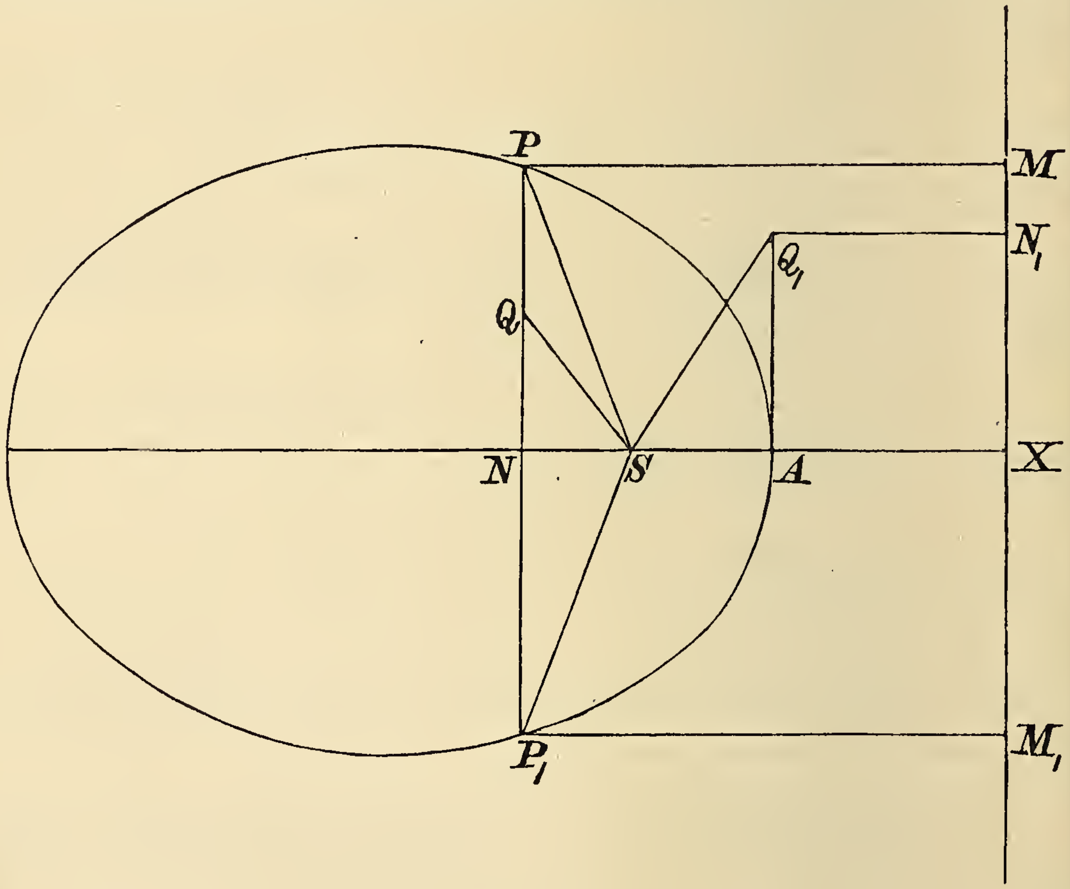
16. The focal chord, $LSL_,$, drawn perpendicular to th major axis, is called the *Latus Rectum*.

17. The circle, described on the major axis as di meter, is called the *Auxiliary Circle*.

## Prop. III.

*Chords perpendicular to the major axis are bisected b the major axis.*

Let $PNP_,$ a chord perpendicular to $AA_,$ meet it in N



$$PN = P_,N.$$

Join $SP$, $SP_,$, and draw $PM$, $P_,M_,$ perpendicular t the directrix.

Then, $PM_{,}$ being a parallelogram,

$$PM = P_{,}M_{,}.$$

But $SP : SP_{,} :: PM : P_{,}M_{,}$;

$$\therefore\ SP = SP_{,}.$$

But $PNS$ is a right angle;

$$\therefore\ PN = NP_{,}. \qquad \text{[L. 2. Cor.}$$

Therefore chords, &c. Q. E. D.

N.B. The chord $PP_{,}$ is sometimes called the double rdinate.

Cor. 1. From this it is evident that no chord parallel the directrix can cut the curve in more points than vo. For if possible, let $Q$, a point in $PN$, lie on the rve;

$$\therefore\ QN = NP_{,} = NP, \text{ which is impossible.}$$

Cor. 2. The perpendicular to the major axis through ither of its vertices cannot meet the curve again.

## Prop. IV.

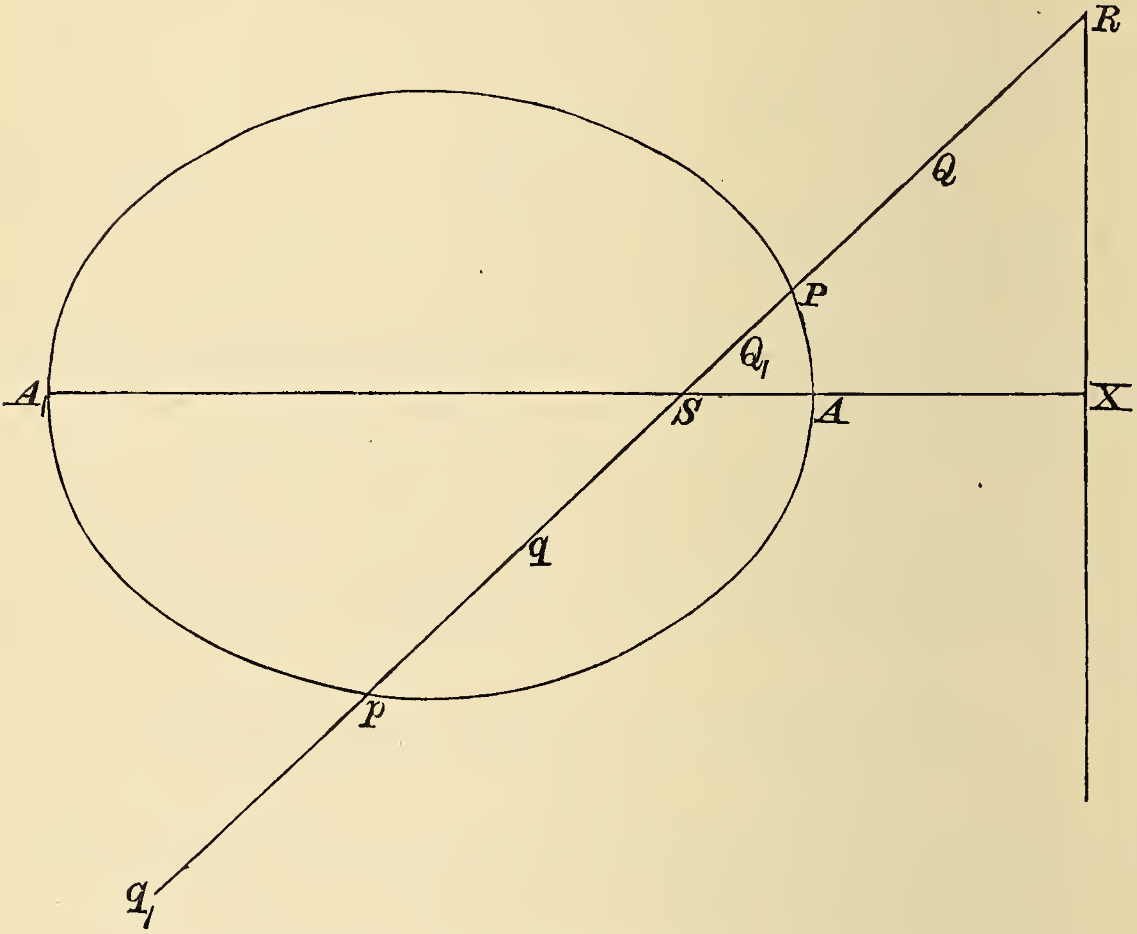
*If a point lie without an ellipse, the ratio of its distances rom the focus and directrix is greater than e* : 1; *if with-, less than e* : 1.

Let $Qq$ be external points, $Q_{,}q_{,}$ internal points, on e focal chord $Pp$; $R$ the foot of the chord.

Then $Qq$ lying outside $P$ and $p$ respectively,

$SQ : QR$ is $> SP : pR$, [L. 7. Cor. 1.

and $Sq : qR$ is $> Sp : pR$; [L. 7. Cor. 1.



therefore the ratio of the distances of $Q$ or $q$ from the focus and directrix is greater than the ratios of the distances of $P$ or $p$, [1. Cor. 2.

that is $> e : 1$. [D. 1.

Similarly, the ratio of the distances of $Q_{,}q_{,}$ from the focus and directrix is less than that of the distances of $P$, $p$, and therefore less than $e : 1$.

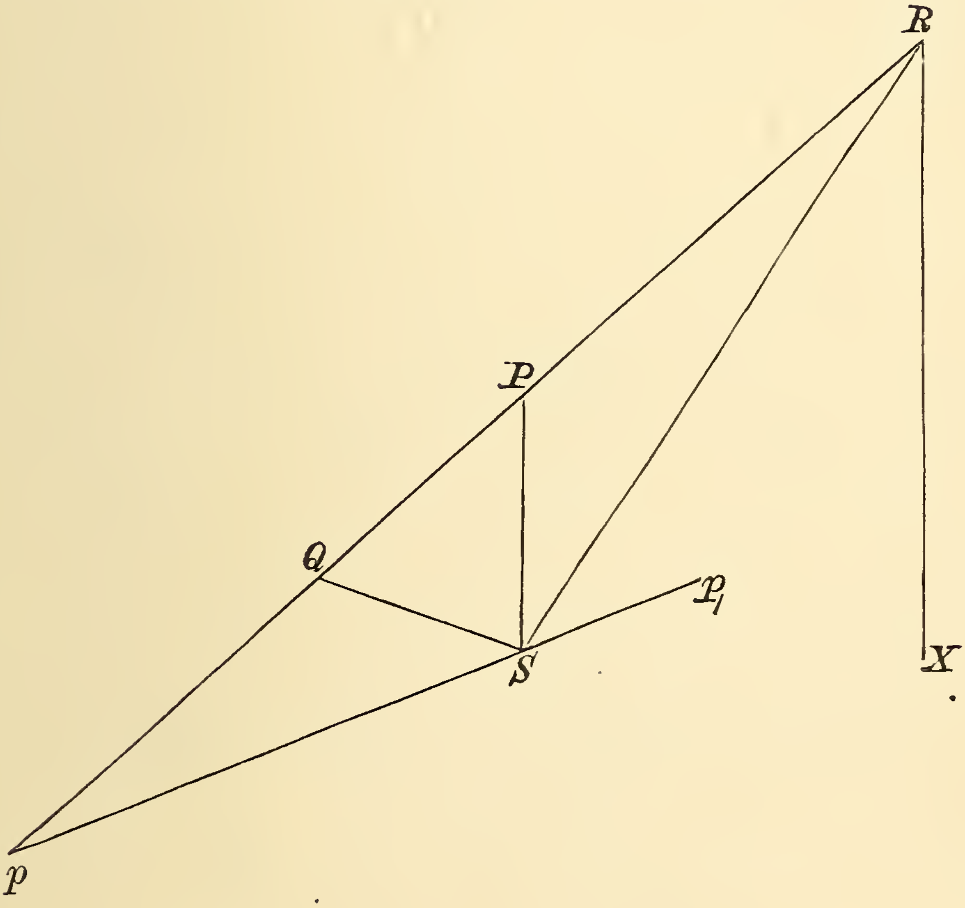
Hence, if a point, &c. Q. E. D.

## Prop. V.

*No straight line can meet the curve in more points than two.*

For, if it be possible, let a chord $PQp$ meet the curve in $P$, $Q$, $p$.

Produce $PQp$ to meet the directrix in $R$.



Join $SR$, $SP$, $SQ$, $Sp$, and produce $pS$ to $p_{\prime}$.

Then $Sp : SQ :: pR : QR$; [I. Cor. I.

$\therefore \angle QSR = RSp_{\prime}$. [E. VI. A.

Similarly, $\angle PSR = RSp_{\prime}$;

$\therefore \angle QSR = PSR$;

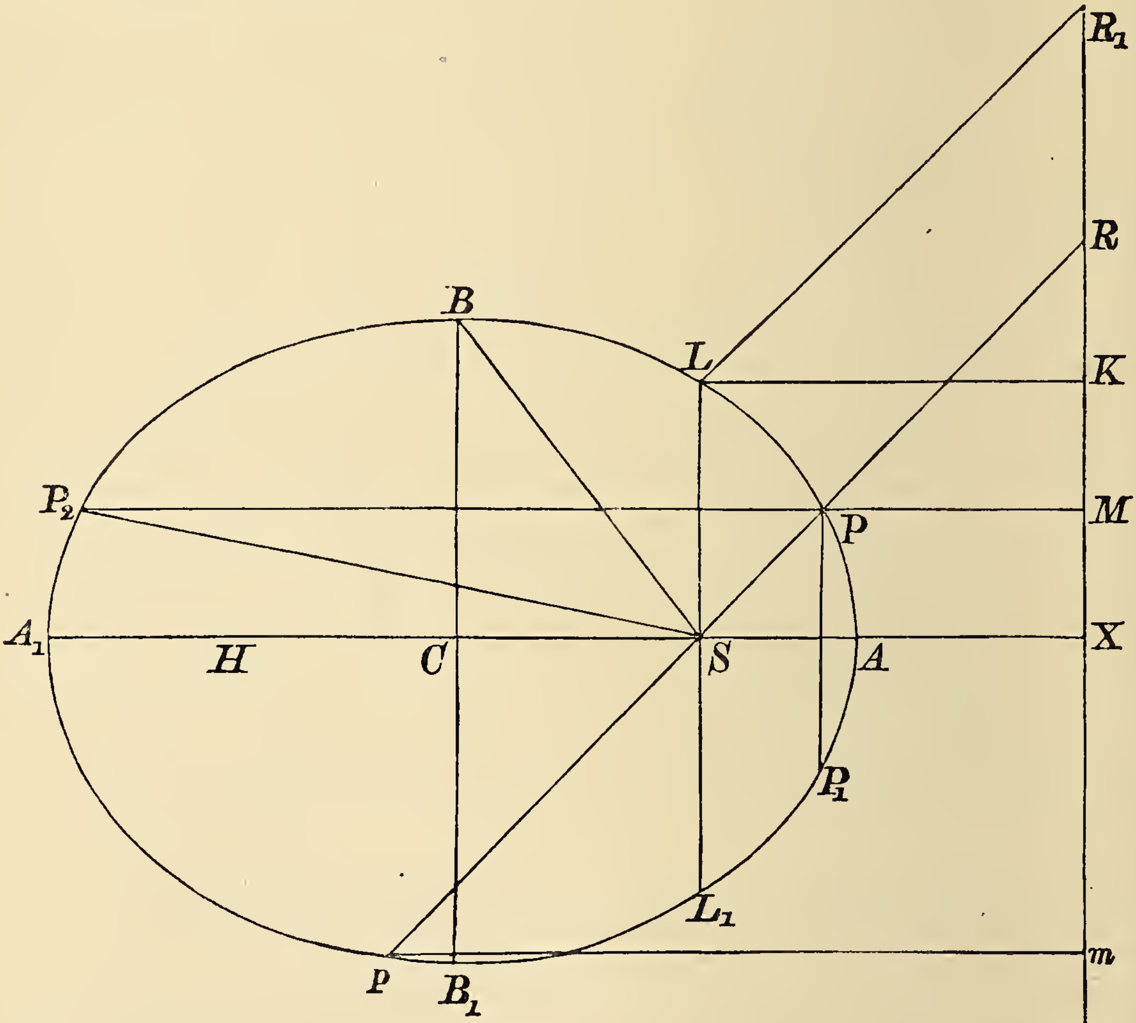
which is impossible.

But, if $PQp$, when produced, do not meet the directrix, $PQp$ is perpendicular to the axis major, and cannot meet the curve in more points than two. [3. COR. 1.

Hence, no straight line, &c. Q. E. D.

## PROP. VI.

*If $X$ be the foot of the major axis, and $B$ an extremity of the minor axis;*



$$CS : CA :: CA : CX :: e : 1;$$

and $$SB = CA.$$

For, since $A$, $A_{,}$ are points on the curve,

$$SA : AX :: e : 1,$$ [D. 1.

and $SA_{,} : A_{,}X :: e : 1$; [D. 1.

therefore the points $A$, $A_{,}$ divide $SX$ internally and externally in the ratio $e : 1$.

But $C$ is the bisection of $AA_{,}$. [D. 8.

Hence $CS : CA :: e : 1$, [L. 8.

and $CA : CX :: e : 1$; [L. 8.

$$\therefore CS : CA :: CA : CX :: e : 1.$$

Again, $SB : CX :: e : 1$, [D. 1.

$$:: CA : CX;$$

$$\therefore SB = CA.$$

Therefore, if $X$, &c. Q. E. D.

COR. 1. $CS : CX :: e^2 : 1$.

COR. 2. $CS \,.\, CX = CA^2$. [E. VI. 16.

## PROP. VII.

*The semiaxis minor is a mean proportional between the parts into which the focus divides the major axis.*

$$BC^2 = AS \,.\, SA_{,}.$$

For $SB^2 = CA^2$ [6.

$= AS \,.\, SA_{,} + CS^2$. [E. II. 5.

But $SB^2 = BC^2 + CS^2$; [E. I. 47.

$\therefore BC^2 = AS \,.\, SA_{,}$. Q. E. D.

COR. 1. $CS \,.\, SX = BC^2$.

For $CS \,.\, SX = AS \,.\, SA_{,}$ [L. 8. Cor. 2.

$= BC^2$.

COR. 2. $SL$ is a third proportional to $CA$, $BC$.

For $SL : SX :: e : 1$ [D. 1.

$:: CS : CA$; [6.

$\therefore CA . SL = CS . SX$

$= BC^2$; [COR. 1.

$\therefore CA : BC :: BC : SL$. Q. E. D.

COR. 3. $SX : CX :: BC^2 : CA^2$.

For $CS . SX = BC^2$, [COR. 1.

and $CS . CX = CA^2$; [6. COR. 2.

$\therefore SX : CX :: BC^2 : CA^2$.

N.B. In the usual notation, $CA$ is represented by $a$, $CB$ by $b$;

$$\therefore CS = ea; \quad CX = \frac{a}{e}; \quad \text{and } b^2 = (1 - e^2)a^2.$$

## PROP. VIII.

*Four times the rectangle contained by the parts of a focal chord is equal to the rectangle contained by the whole chord and the latus rectum.*

Let $PSp$ be any focal chord, $R$ its foot.

$$LL_, . Pp = 4SP . Sp.$$

Since $SP : PR :: Sp : pR$; [1. COR. 1.

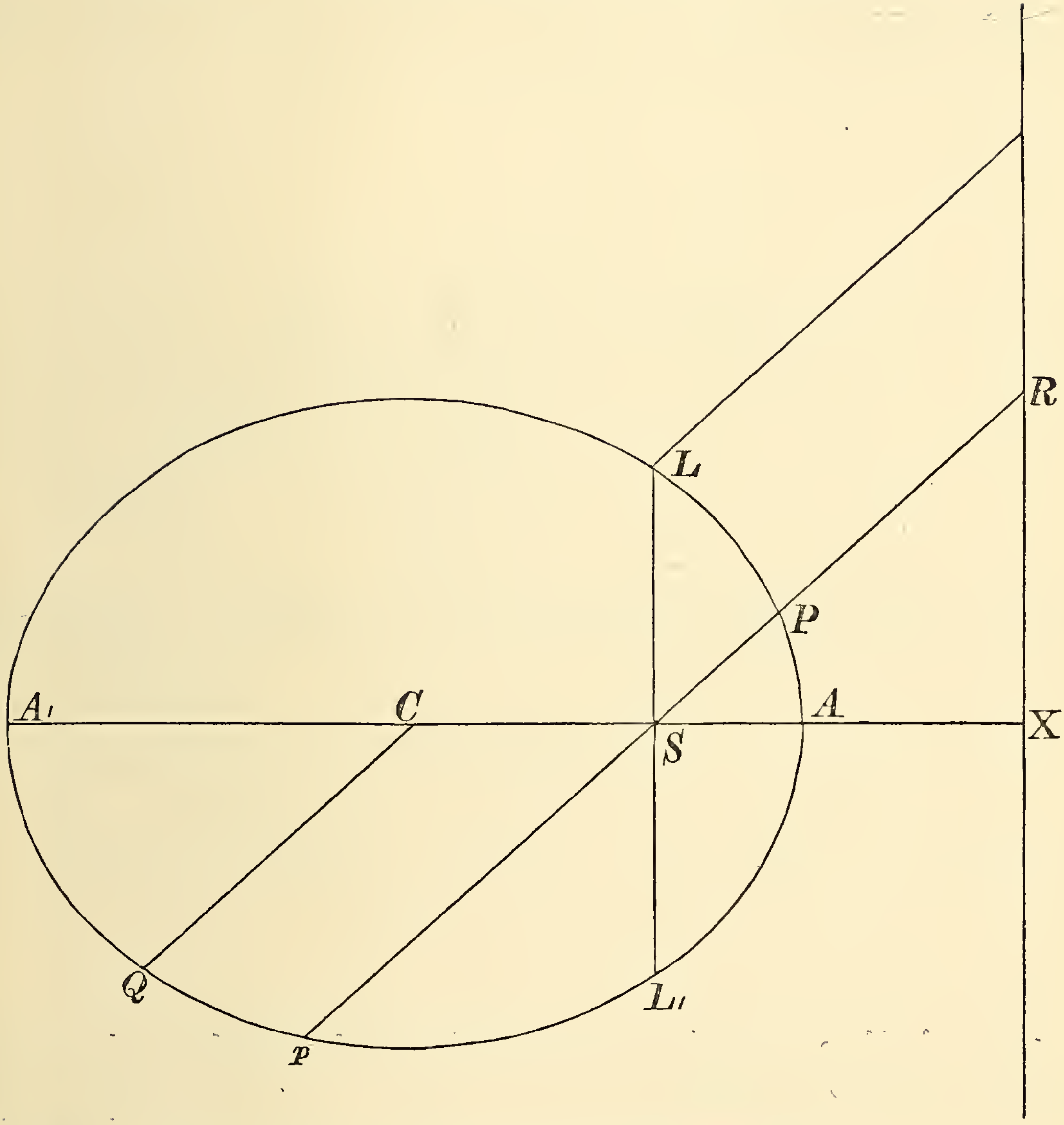
$\therefore SP . Sp : \frac{1}{2}Pp . SR :: Sp : pR$ [L. 8. COR. 4.

$:: \frac{1}{2}SL : SR$ [1.

$:: \frac{1}{2}LL_, : SR$; [3.

$$\therefore\ SP \cdot Sp = \tfrac{1}{4} Pp \cdot LL_{,}$$

and $$4SP \cdot Sp = Pp \cdot LL_{,}.$$ Q. E. D.



N.B. This result is often thus expressed;

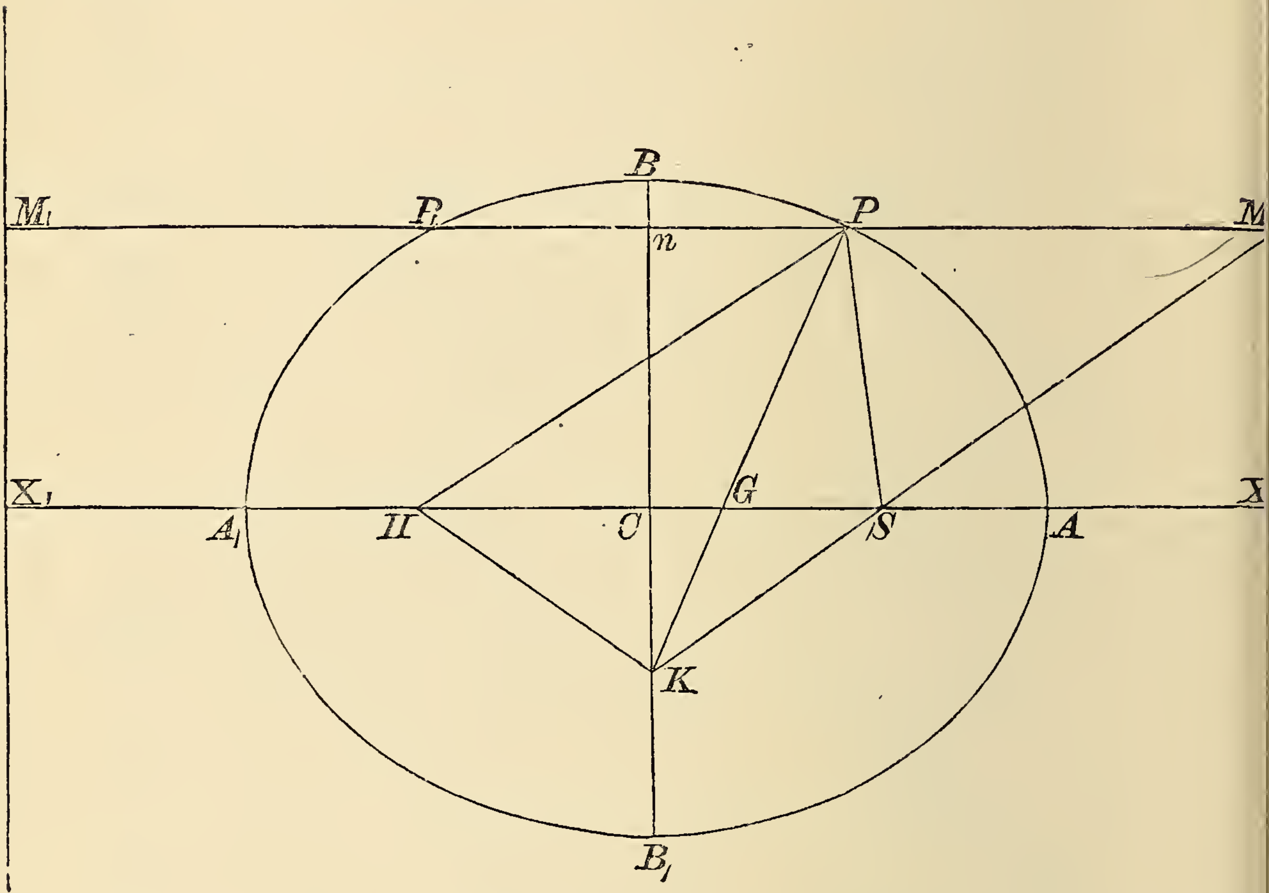
$$\frac{1}{SP} + \frac{1}{Sp} = \frac{2}{SL}.$$

For $$\frac{Sp + SP}{Sp \cdot SP} = \frac{2}{SL};$$

$$\therefore\ \frac{1}{SP} + \frac{1}{Sp} = \frac{2}{SL}.$$

## DEFINITIONS.

18. If a point $H$ be taken in the major axis, so that $SC = CH$, $H$ is called the *Conjugate Focus.*



19. A line, parallel to the directrix, and at the same distance from the centre, is called the *Conjugate Directrix.*

## PROP. IX.

*The sum of the focal distances of any point on the curve is constant, and equal to the major axis,*

$$SP + PH = AA_{,}.$$

Draw $PM$ perpendicular to the directrix. Join $MS$, meeting the minor axis in $K$, and $PK$, meeting the major axis in $G$.

Also join $HK$.

Then because $PM$ is parallel to $CX$,

$GS : PM :: KS : KM$ [E. VI. 4.

$:: CS : CX$ [E. VI. 4.

$:: e^2 : 1$ [6. Cor. 1.

$:: SP^2 : PM^2$; [D. 1.

$\therefore GS : SP :: SP : PM.$

But the $\angle GSP = SPM$, being alternate angles;

$\therefore \angle SPK = PMS$ [E VI. 6.

$= HSK$

$= KHS$;

therefore a circle can be described about the quadrilateral $SPHK$. [E. III. 23.

But the chords of this circle, $KS . KH$, are equal.

Hence the subtended angles, $KPS$, $KPH$ are equal. [E. III. 28.

$\therefore HP : HC :: SP : SG,$ [E. VI. 3.

or $HP + SP : HG + SG :: SP : SG$

$:: PM : SP$

$:: 1 : e$ [D. 1.

$:: AA_{,} : SH$; [6.

$\therefore HP + SP = AA_{,}$. Q. E. D.

Cor. 1. From the course of the proof it is evident that the triangles $KPM$, $KSP$ are similar;

$\therefore KP^2 = KS . KM.$ [E. VI. 4.

Cor. 2. The minor axis bisects all chords parallel to the major axis.

For, produce $MP$ to meet the curve in $P_{\prime}$, and let $PP_{\prime}$ cut the minor axis in $n$.

$KP^2 = KS . KM.$ [Cor. 1.

Similarly, $KP_{\prime}^2 = KS . KM$;

$\therefore KP = KP_{\prime}$;

$\therefore Pn = P_{\prime}n$, [L. 2. Cor.

or $PP_{\prime}$ is bisected in $n$. Q. E. D.

Cor. 3. From this it is evident that the curve is symmetrical with regard to the minor axis, as well with regard to the major axis; [3.

Hence the two axes will divide the curve into four equal and every way similar quadrants.

Also, since any lines through the centre equally inclined to the major axis in these respective quadrants will be equal, it will follow that every diameter is bisected at the centre, and that diameters equally inclined to the major axis are equal.

Cor. 4. $PG . PK = HP . SP.$

For, the triangles $KPS$, $HPG$ are similar;

$\therefore PK : SP :: HP : PG$; [E. vi. 4.

$\therefore PG . PK = HP . SP.$

Cor. 5. $KG : KP :: e^2 : 1$,

and $PG^2 : SP, HP :: BC^2 : CA^2$.

For $KG : KP :: KS : KM$

$:: CS : CX$

$:: e^2 : 1.$ [6. Cor. 1.

Also $KG : KP :: CS : CX$;

$$\therefore PG : KP :: SX : CX$$

$$:: BC^2 : CA^2; \qquad \text{[7. Cor. 3.}$$

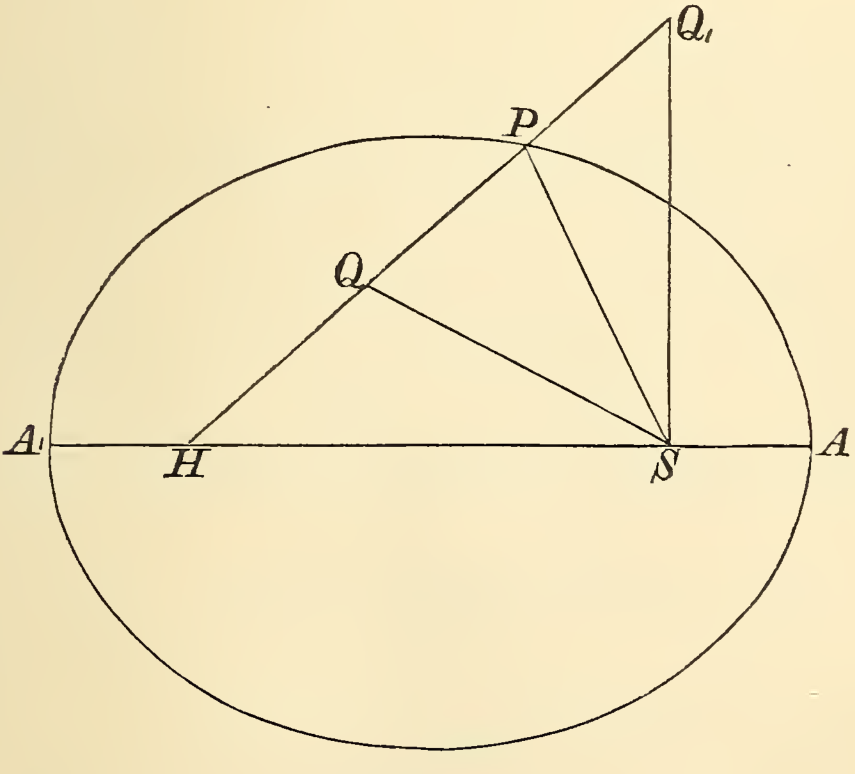
$$\therefore PG^2 : KP.PG :: BC^2 : CA^2,$$

$$PG^2 : HP.SP :: BC^2 : CA^2. \qquad \text{[Cor. 4.}$$

### Prop. X.

*The sum of the focal distances of any point exterior to the ellipse is greater than the major axis; of any point interior, less.*

Let a line $HP$ meet the ellipse in $P$, and let $Q$, $Q_{\prime}$ be an interior and exterior point respectively.



Then $SQ + HQ$ is $< SP + HP$, [E. I. 21.

and $SQ_{\prime} + HQ_{\prime}$ is $> SP + HP$. [E. I. 21.

But $$SP + HP = AA_{,};$$ [9.

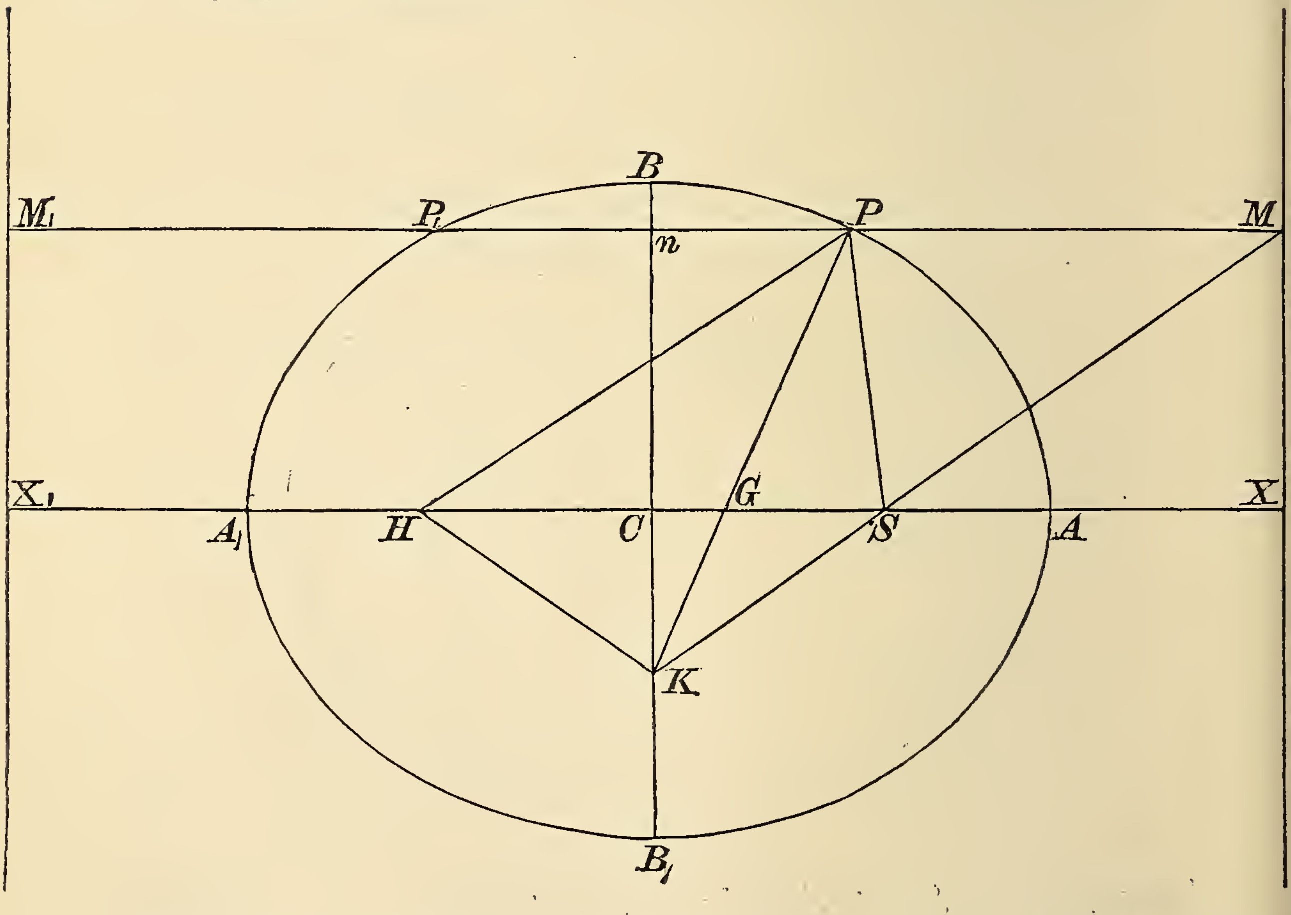
$$\therefore\ SQ + HQ < AA_{,},$$

and $$SQ_{,} + HQ_{,} > AA_{,}.$$ Q. E. D.

Cor. From this it is evident that if an inextensible string have its ends fixed at $S$ and $H$, and be kept stretched by a pencil at $P$, the pencil will describe an ellipse, whose major axis is equal to the length of the string, and whose foci are $S$ and $H$.

## Prop. XI.

*The curve may be described with the conjugate focus and conjugate directrix.*



Let $P$ be any point on the curve. Draw $MPM_{,}$ perpendicular to the directrices. Join $SP$. $HP$.

Then $SP + HP = 2CA$. [9.
But $MP + PM_{,} = 2CX$;
$\therefore SP + HP : MP + PM_{,} :: CA : CX$
$:: e : 1$ [6.
$:: SP : PM$; [D. 1.

therefore also $HP : PM_{,} :: e : 1$

or the ellipse may be described with $H$ and $X_{,}M_{,}$ as focus and directrix, and with the same excentricity. Q. E. D.

Cor. If, through $P$, any line $RPR_{,}$ be drawn to the directrices,

$$SP : HP :: PR : PR_{,}.$$

## Prop. XII.

*The foot of any chord is centre of similitude of circles described through the focus, with the extremities of the chords as centres. And the tangents to these circles drawn from this centre of similitude will be parallel to the tangents to the auxiliary circle, drawn through the foot of the diameter to which the chord is parallel.*

Let $R_{,}$ be the foot of the chord $QQ_{,}$, and $R$ of the parallel diameter.

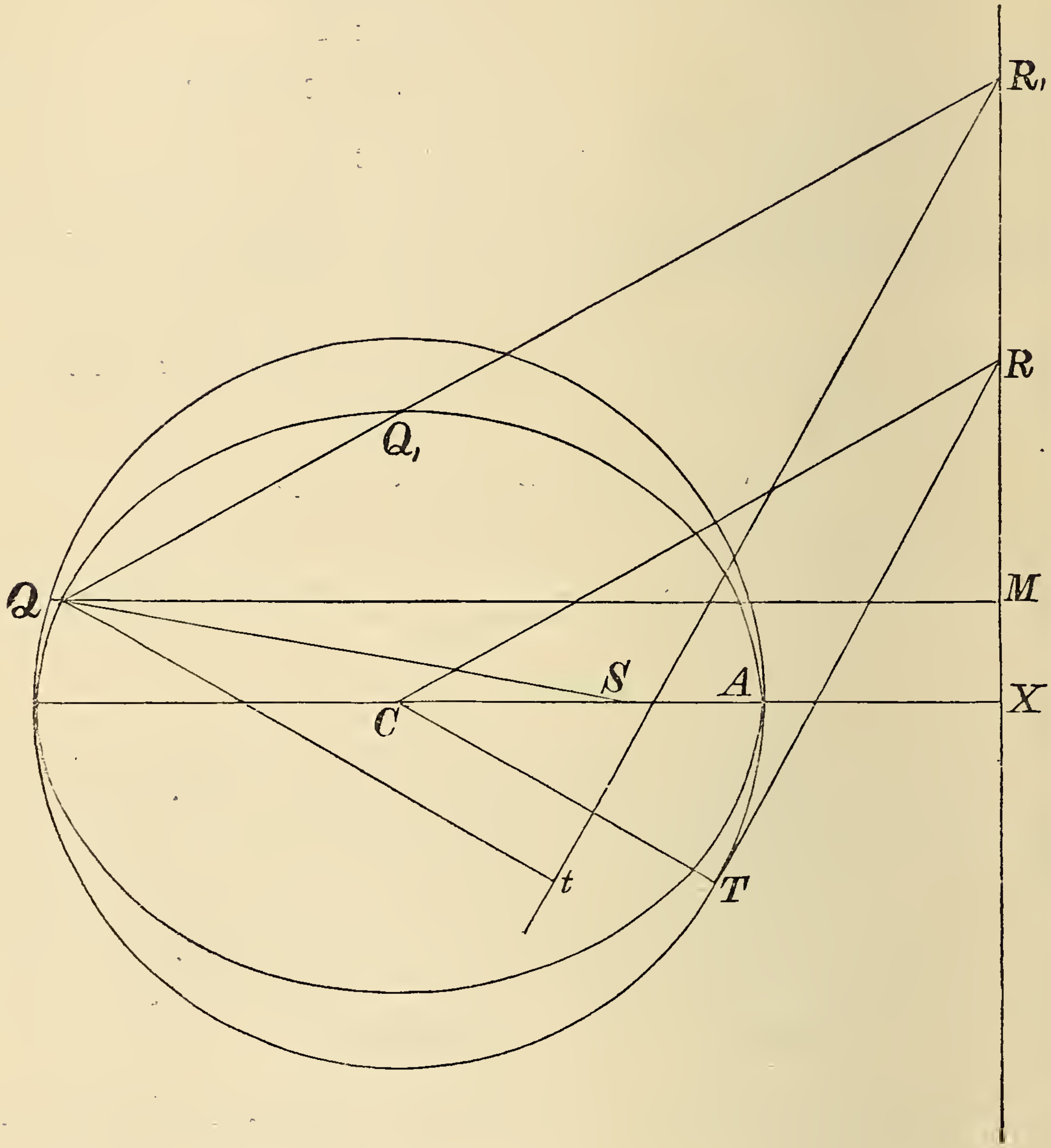
Draw $RT$ a tangent to the auxiliary circle, and $R_{,}t$ parallel to $RT$.

Join $CT$, and draw $Qt$ perpendicular to $R_{,}t$, and $QM$ perpendicular to the directrix.

Then $CA : CX :: e : 1$ [6.
$:: SQ : QM$; [D. 1.

$$\therefore CA : CR :: SQ : QR_{,} ;$$ [E. VI. 4.

$$\therefore CT : CR :: SQ : QR_{,}.$$



But by similar triangles

$$CT : CR :: Qt : QR_{,}.$$

Hence $$SQ = Qt;$$

and the circle described through $S$ with $Q$ as centre will pass through $t$, and touch $R_{,}t$.

Similarly, the circle described from $Q_{,}$ with radius $Q_{,}S$, will touch $R_{,}t$.

And, in the same manner, they will touch the line drawn through $R_{,}$, parallel to the other tangent drawn from $R$ to the auxiliary circle.

Hence $R_{,}$ will be centre of similitude of the two circles, and the tangents from $R_{,}$ will be parallel, &c. Q. E. D.

COR. Exactly the same proof would shew that if we take any other point in $QQ_{,}$ as centre, and describe a circle through the point which divides the perpendicular to the directrix therefrom in the ratio $e : 1 - e$, this circle will also touch $R_{,}t$, and the other tangent through $R_{,}$.

## PROP. XIII.

*Parallel chords are bisected by the diameter passing through the intersection of the directrix with the line through the focus perpendicular to the chords.*

Let $QQ_{,}$ be a chord parallel to the diameter $CP$; $R_{,}R$ the feet of the chord and diameter; $SG$ the focal chord perpendicular to them. $QQ_{,}$ will be bisected by the diameter through $G$.

Bisect $QQ_{,}$ in $m$.

Let $GS$ intersect the chord and diameter in $g_{,}g$, and meet in $p$ the tangent $Rtt_{,}$ to the circles through $S$ with centres $QQ_{,}$. [12.

Also from $R$ draw $RT$ parallel to $Rtt_{,}$, touching the auxiliary circle in $T$. Join $pm$.

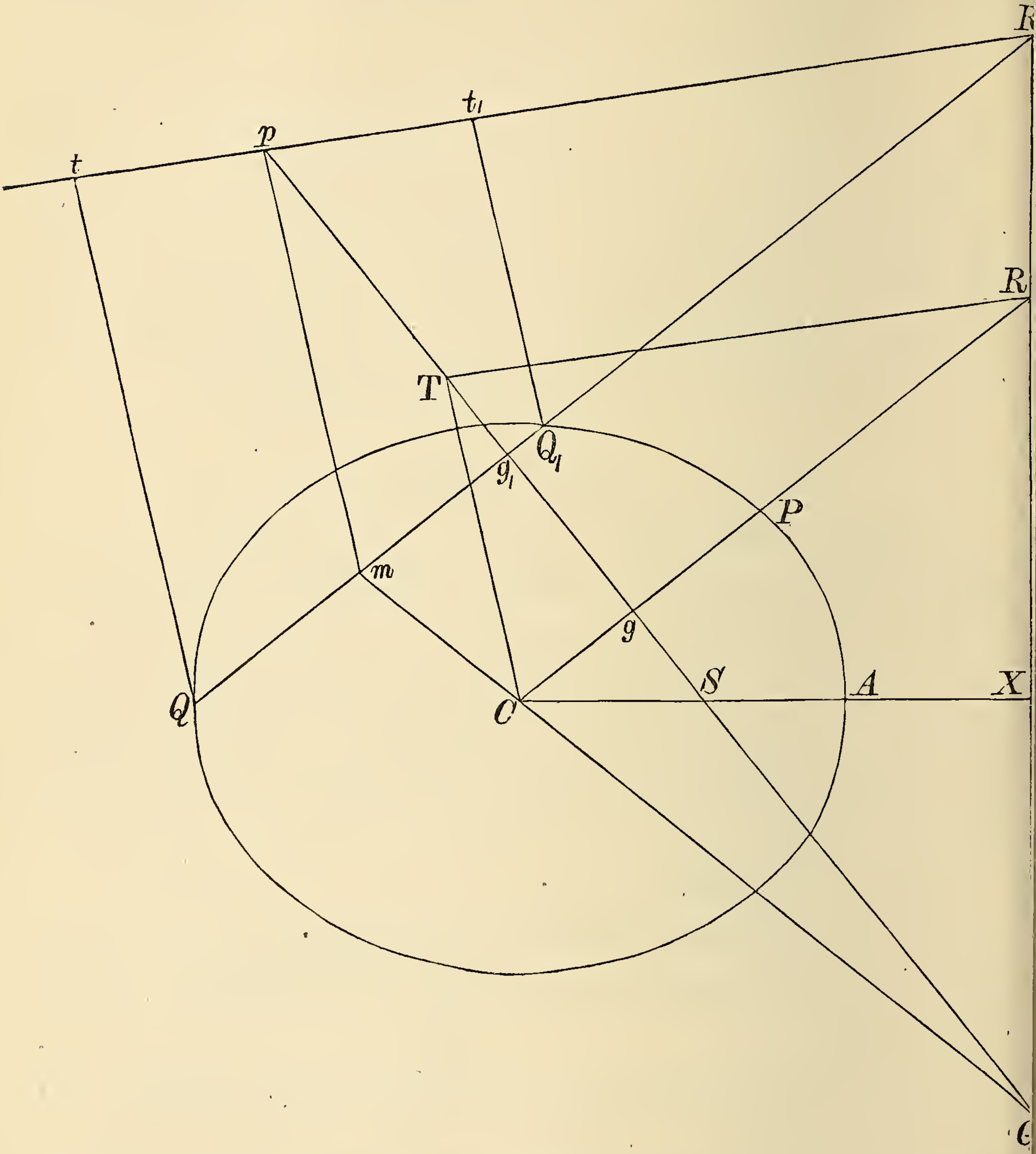
Then $CS \,.\, CX = CA^2$. [6. COR. 1.

Therefore $S$ is the pole of $RX$ with regard to the auxiliary circle; [L. 10. COR. 2.

and the polar of $R$ passes through $S$, and is perpendicular to $CR$. [L. 11. COR. 3.

Hence $SG$ is the polar of $R$, and passes through $T$.

Now $SG$ passing through $S$ and perpendicular to $QQ_1$ will evidently be the common chord of the circles described through $S$ with centres $QQ_1$.



But $tt_1$ is the common tangent;

∴ $SG$ bisects $tt_1$, and $pt = pt_1$. [L. 4.

But $Qm = Q_1m$;

∴ $mp$ is parallel to $Qt$, $Q_1t_1$; [L. 3. Cor. 2.

$\therefore\ mpR_{,}$ is a right angle.

Hence the similar triangles $mPR$, $CTR$ are similarly divided by $pg_{,}$, $Tg$;

$\therefore\ mg_{,} : g_{,}R_{,} :: Cg : gR$. [L. 1.

Hence the three lines $mC$, $g_{,}g$, $R_{,}R$ pass through one point; or $m$ lies on the diameter $CG$. Q. E. D. [L. 3.

COR. 1. The line joining the bisections of two parallel chords passes through the centre.

For this line will be the bisecting diameter.

COR. 2. The diameter bisecting any chord will bisect any other parallel chord.

COR. 3. If two chords are bisected by the same diameter, they are parallel to one another.

For let $CG$ be the bisecting diameter. Then $SG$ is perpendicular to each chord; therefore they are parallel to each other.

## PROP. XIV.

*The rectangle contained by any diameter of the ellipse, and the perpendicular chord of the auxiliary circle which passes through the focus, is equal to the rectangle contained by the axes.*

Let $PCP_{,}$ be any diameter, $R$ its foot; $RT$, $RT_{,}$, tangents to the auxiliary circle.

Draw $PK$ perpendicular to $RT$.

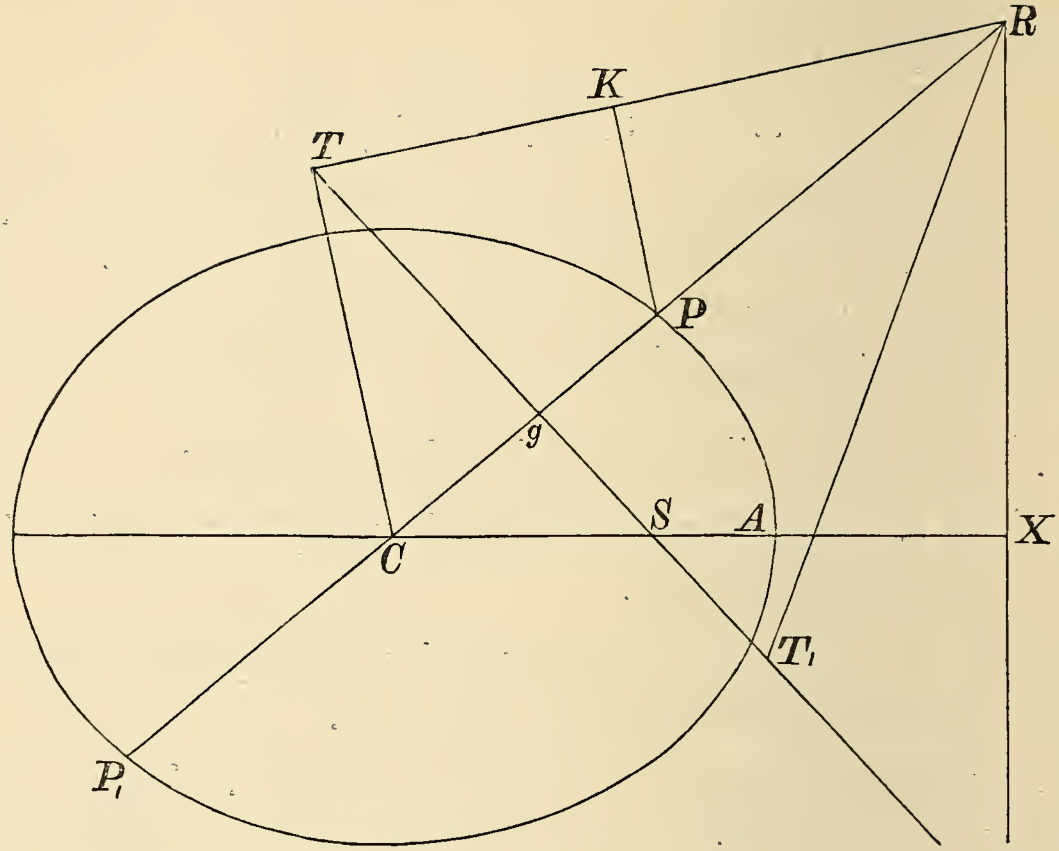
Now $CS \,.\, CX = CA^2$; [6. COR. 2.

$\therefore\ TT_{,}$ passes through $S$ and is perpendicular to $RC$;

[L. 10 and L. 11.

$\therefore\ TT_{,}$ is bisected in $g$. [E. III. 3.

Again, $PK = PS,$ [12.



and $TK^2 = TS \, . \, ST_{,}$ [L. 4. Cor.

$= AS \, . \, SA_{,}$ [E. III. 35.

$= BC^2.$ [7.

But $CP : TK :: CR : RT$

$:: CT : Tg$ [E. VI. 8.

$:: CA : Tg;$

or $PP_{,} : BB_{,} :: AA_{,} : TT_{,};$

$\therefore PP_{,} \, . \, TT_{,} = BB_{,} \, . \, AA_{,}.$ Q. E. D.

Cor. 1. Since the diameters of the ellipse vary inversely as the perpendicular chords of the auxiliary circle through $S$, the minor axis will be the least diameter, being perpendicular to the diameter $AA_{,}$. [L. 9.

Also the major axis will be the greatest, being perpendicular to the shortest chord.

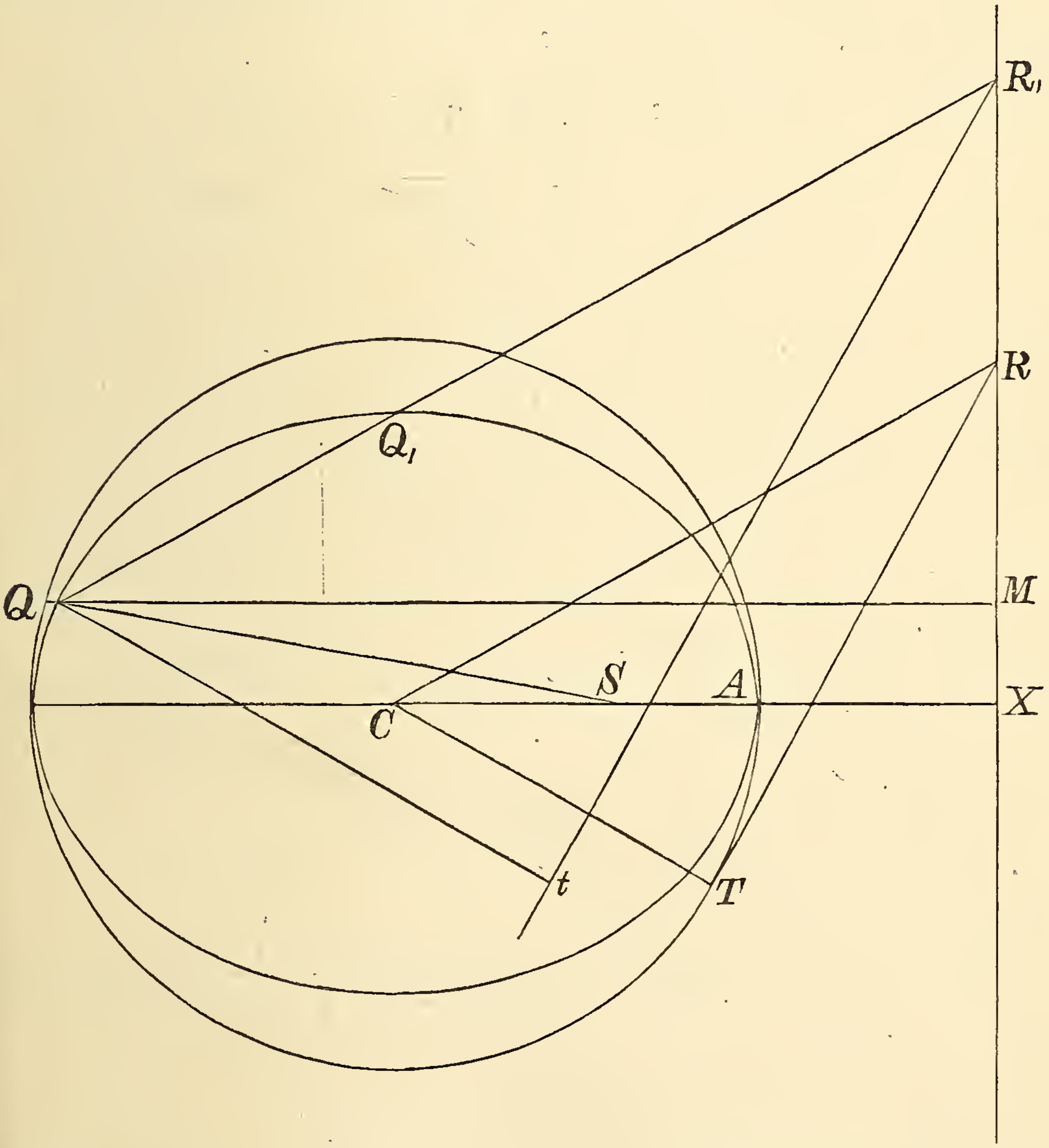
And of the rest, that which is nearer to the major axis will be greater than the more remote.

COR. 2. It is proved in the foregoing proposition that

$$TK = BC.$$

Hence if $RT$ is drawn tangent to the auxiliary circle

$$RT : RC :: BC : CP.$$



Hence also if $R_{,}t$ be a tangent through the foot of a chord $QQ_{,}$, drawn to the circle described from $Q$ to pass through $S$,

$$R_{,}t : R_{,}Q :: BC : CP,$$

where $CP$ is the semi-diameter parallel to $QQ_{,}$.

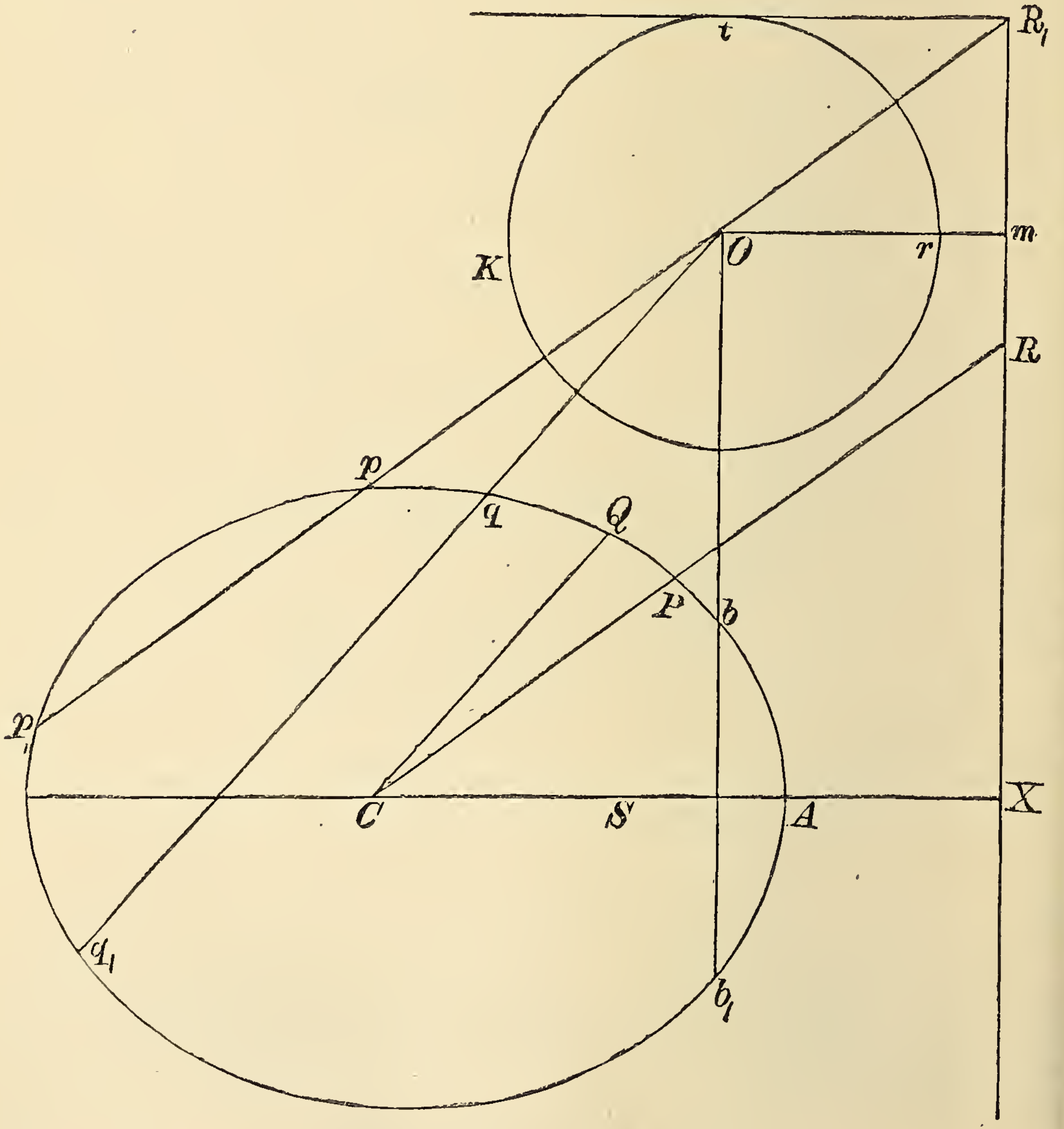
For $$RT : RC :: R_{,}t : R_{,}Q.$$

## PROP. XV.

*The rectangles contained by the segments of secants drawn to an ellipse from an exterior point are in the same ratio as the squares of the parallel semi-diameters.*

Let $Opp_{\prime}$, $Oqq_{\prime}$ be two secants drawn from an exterior point $O$ parallel to $CP$, $CQ$;

$$Op \,.\, Op_{\prime} : Oq \,.\, Oq_{\prime} :: CP^2 : CQ^2.$$



Draw $Om$ perpendicular to the directrix, and divide it in $r$ in the ratio of $e : 1 - e$.

At distance $Or$ describe a circle about $O$, and from $R_{,}$ the foot of the chord $pp_{,}$ draw $R_{,}t$ tangent to the circle.

Then the point $R_{,}$ will be a common centre of similitude of this circle, and of the circles described through $S$ with $p, p_{,}$ as centres. [12. Cor.

Therefore if a tangent from $S$ to the circle $tr$ meet it in $K$, and $SK$ be joined;

$$SK^2 : Op \,.\, Op_{,} :: R_{,}t^2 : R_{,}O^2. \qquad \text{[L. 6. Cor. 2.}$$

But $$R_{,}t^2 : R_{,}O^2 :: BC^2 : CP^2; \qquad \text{[14. Cor. 2.}$$

$$\therefore\ SK^2 : Op \,.\, Op_{,} :: BC^2 : CP^2,$$

and $$Op \,.\, Op_{,} : CP^2 :: SK^2 : BC^2.$$

Similarly, $$Oq \,.\, Oq_{,} : CQ^2 :: SK^2 : BC^2;$$

$$\therefore\ Op,\ Op_{,} : Oq,\ Oq_{,} :: CP^2 : CQ^2.$$

It is likewise evident that if $Ob$, $Ob_{,}$ be drawn parallel to the minor axis, $Ob \,.\, Ob_{,} = SK^2$.

For $$Sb = Sb_{,} = Or,$$

and a circle equal to $rtK$ would be described through $b$, $b_{,}$ with centre $S$;

$$\therefore\ Ob \,.\, Ob_{,} = SK^2;$$

$$\therefore\ Op \,.\, Op_{,} : Ob \,.\, Ob_{,} :: CP^2 : CB^2.$$

Therefore the rectangles &c. Q. E. D.

N. B. It is evident that, $Or$ being less than $SO$, [4. $S$ will lie exterior to the circle.

Cor. Tangents from an external point to an ellipse are in the same ratio as the parallel semi-diameters.

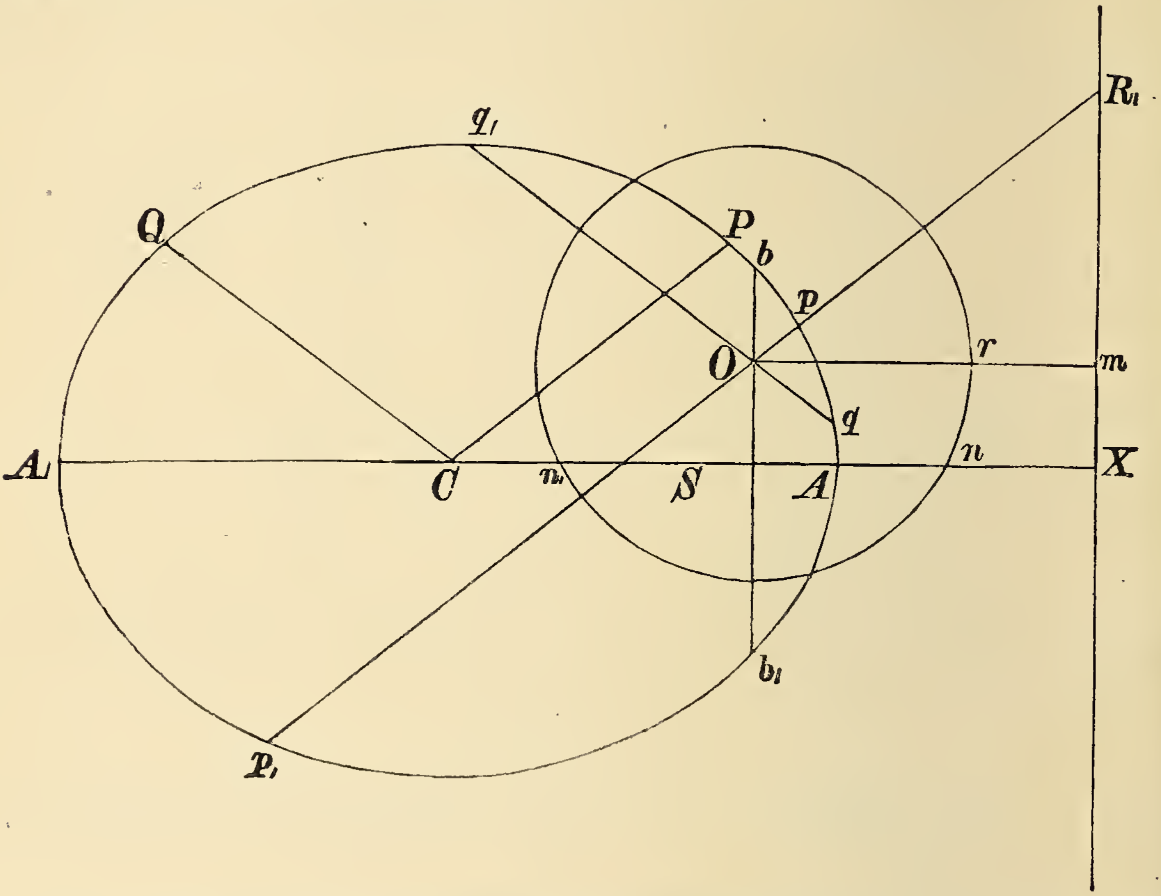
For if the points $q$, $q_{,}$ coincide and also $p$, $p_{,}$, $Op$, $Oq$ become tangents at $p$, $q$;

$$\therefore\ Op^2 : Oq^2 :: CP^2 : CQ^2,$$

$$Op : Oq :: CP : CQ.$$

## PROP. XVI.

*The rectangles contained by the segments of chords, drawn to an ellipse through an interior point, are in the same ratio as the squares of the parallel semi-diameters.*



Let $pOp_{,}$, $qOq_{,}$ be chords, drawn through $O$, parallel to $CP$, $CQ$;

$$pO\,.\,Op_{,} : qO\,.\,Oq_{,} :: CP^2 : CQ^2.$$

Draw $Om$ perpendicular to the directrix, and describe a circle $rnn_{,}$ from $O$, so that

$$Or : Om :: e : 1.$$

Also let $nSn_{,}$ be a chord of this circle through $S$.

Then, $R_{,}$ is a centre of similitude to the circle $rnn_{,}$, and to the two circles described through $S$ with $p$, $p_{,}$ as centres. [12. COR.

Therefore $nS\,.\,Sn_{,} : pO\,.\,Op_{,} :: BC^2 : CP^2$.

[L. 6. COR. 2 and 14. COR. 2.

Also $$nS\,.\,Sn_{,} : qO\,.\,Oq_{,} :: BC^2 : CQ^2;$$

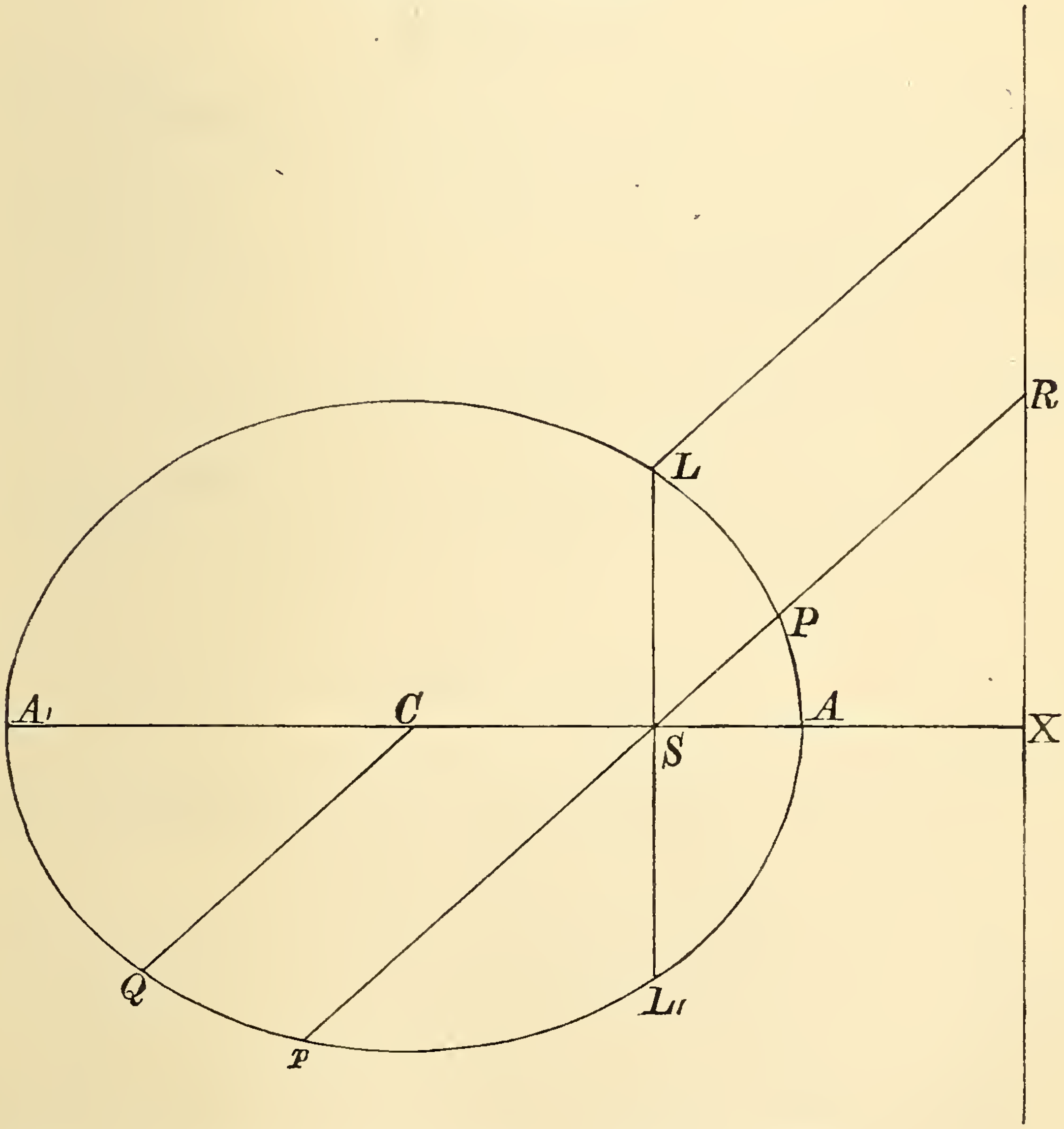
$$\therefore pO\,.\,Op : qO\,.\,Oq_{,} :: CP^2 : CQ^2.$$ Q. E. D.

∴ The rectangles &c.

N.B. It is evident that $S$ will be within this circle. [4.

COR. If $Pp$ be a focal chord parallel to the diameter $Q_{,}CQ$;

$$Pp\,.\,AA_{,} = QQ_{,}^2.$$



For $$SP\,.\,Sp : AS\,.\,SA_{,} :: CQ^2 : CA^2;$$

$$\therefore SP, Sp : CQ^2 :: BC^2 : CA^2$$ [7.

$$:: LL_{,} : AA_{,};$$ [7. COR. 2.

$$\therefore LL_{,}\,.\,Pp : 4CQ^2 :: LL_{,} : AA_{,};$$ [8.

$$\therefore Pp \,.\, AA_{\prime} = 4CQ^2$$
$$= QQ_{\prime}^2. \quad \text{Q. E. D.}$$

N.B. The focal chords are sometimes called parameters to the diameters to which they are parallel, and Props. 15 and 16 are sometimes stated thus:

At any point the rectangles under the segments of any chord or secant vary as the parallel parameters.

## DEFINITIONS.

20. The diameter parallel to the chords bisected by a given diameter is called the *Conjugate Diameter.*

21. The half of the bisected chord is called the *Ordinate* of its extremity.

22. The parts into which a diameter is divided by the ordinate of a point are called the *Abscissæ.*

23. Chords joining any point on a curve to the extremities of a diameter are called *Supplemental Chords.*

## PROP. XVII.

*If one of two supplemental chords is parallel to a diameter, the other is parallel to its conjugate.*

Let $PCP_{\prime}$ be any diameter, $PQ$, $P_{\prime}Q$ any two supplemental chords. If $PQ$ is parallel to any diameter $CM$, $P_{\prime}Q$ will be parallel to the diameter conjugate to $CM$.

Draw $CN$ the conjugate diameter to $CM$, and let $CM$ cut $QP_{\prime}$ in $m$.

Then because $CP = CP_{\prime}$, and $CM$ is parallel to $PQ$;

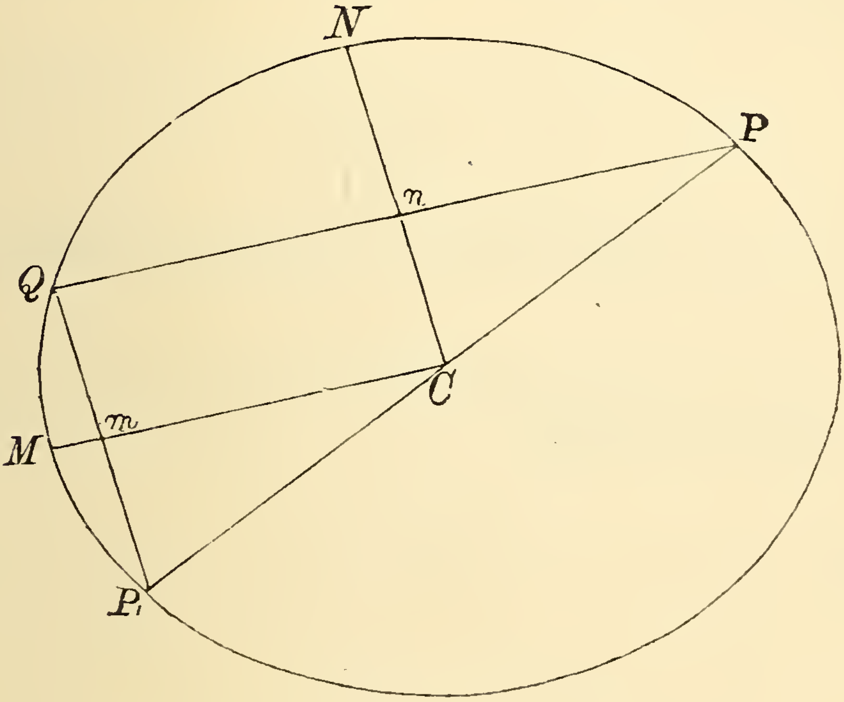
[9. COR. 3.

$$\therefore P_{\prime}m = mQ.$$

Hence $CM$ is the diameter bisecting $QP_{,}$;

$\therefore$ $QP_{,}$ is parallel to $CN$; [D. 20.

that is, $QP_{,}$ is parallel to the conjugate diameter. Q. E. D.



COR. I. If $CN$ is conjugate to $CM$,

$CM$ will also be conjugate to $CN$.

For, through the extremity of any diameter $PP_{,}$, draw $P_{,}Q$ parallel to $CN$. Join $PQ$.

$\therefore$ $P_{,}Q$ is bisected by $CM$; [D. 20.

$\therefore$ $PQ$ is parallel to $CM$. [E. VI. 2.

But $CN$ is parallel to $P_{,}Q$ and passes through the middle point of $PP_{,}$;

$$\therefore Pm = mQ.$$

Hence $CN$ is the diameter bisecting $PQ$; therefore $CM$, which is parallel to $PQ$, is conjugate to $CN$. Q. E. D.

[D. 20.

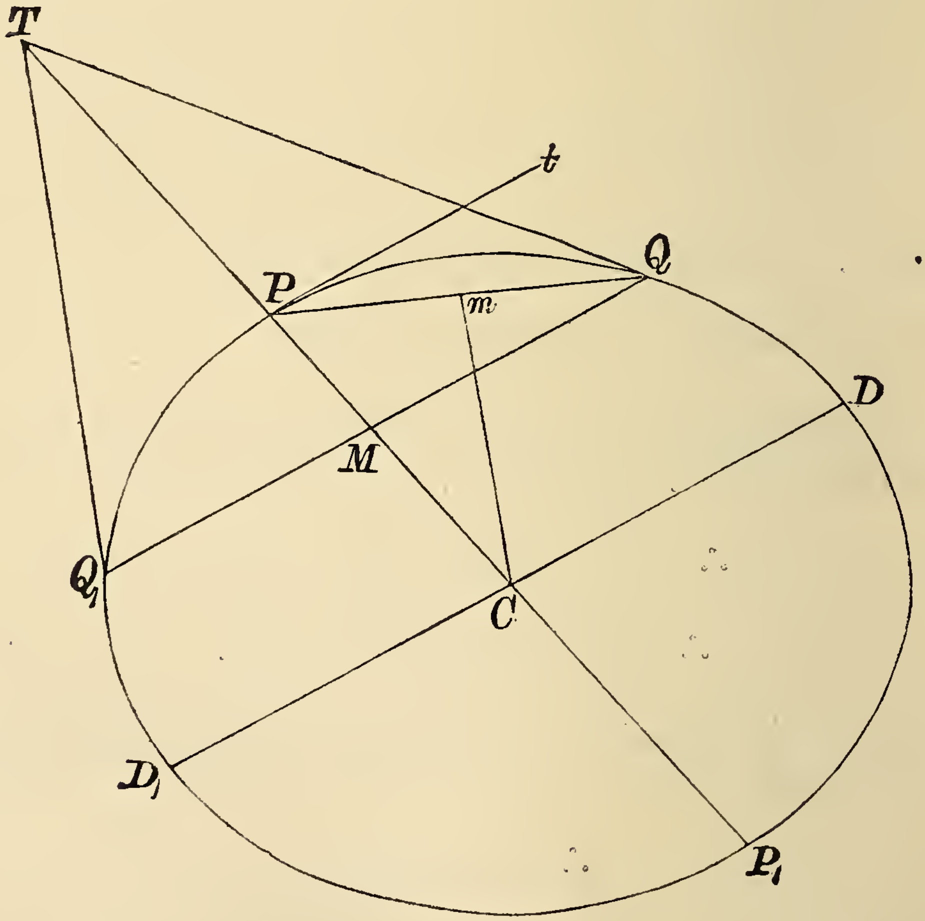
N.B. The relation of conjugate diameters to one another is mutual, and we speak of two conjugate diameters. Also the axes are evidently conjugate diameters.

Cor. 2. The diameters bisecting supplemental chords are conjugate diameters.

## Prop. XVIII.

*Tangents at the extremities of a diameter are parallel to the conjugate diameter.*

Let $PCP_1$ be any diameter; the tangent at $P$ is parallel to the conjugate diameter.



Take any point $Q$ near $P$. Draw $Cm$ bisecting $PQ$.

Then, $PQ$ being bisected by $Cm$,

Any other chord parallel to $PQ$ is bisected by $Cm$. [13. COR. 2.

Let $Q$ move up to coincide with $P$.

Therefore $Cm$ will in direction ultimately coincide with $CP$.

But $PQ$ will become the tangent at $P$.

Hence any chord parallel to the tangent at $P$ will be bisected by $CP$.

Therefore the tangent at $P$ is parallel to the [D. 20.
conjugate diameter. Q. E. D.

COR. 1. Tangents at the extremities of the axes are perpendicular to them.

COR. 2. The line joining the points of contact of parallel tangents is a diameter.

## PROP. XIX.

*Tangents at the extremities of a chord meet in the bisecting diameter.*

Let $QQ_,$ be any chord of an ellipse, $m$ its middle point. Tangents at $QQ_,$ meet in $Cm$.

For $Cm$ will bisect any chord of the ellipse [13. COR. 2.
parallel to $QQ_,$.

Therefore tangents at the extremities of any [L. 12. COR.
of these chords meet in $Cm$.

Hence tangents at the extremities &c. Q. E. D.

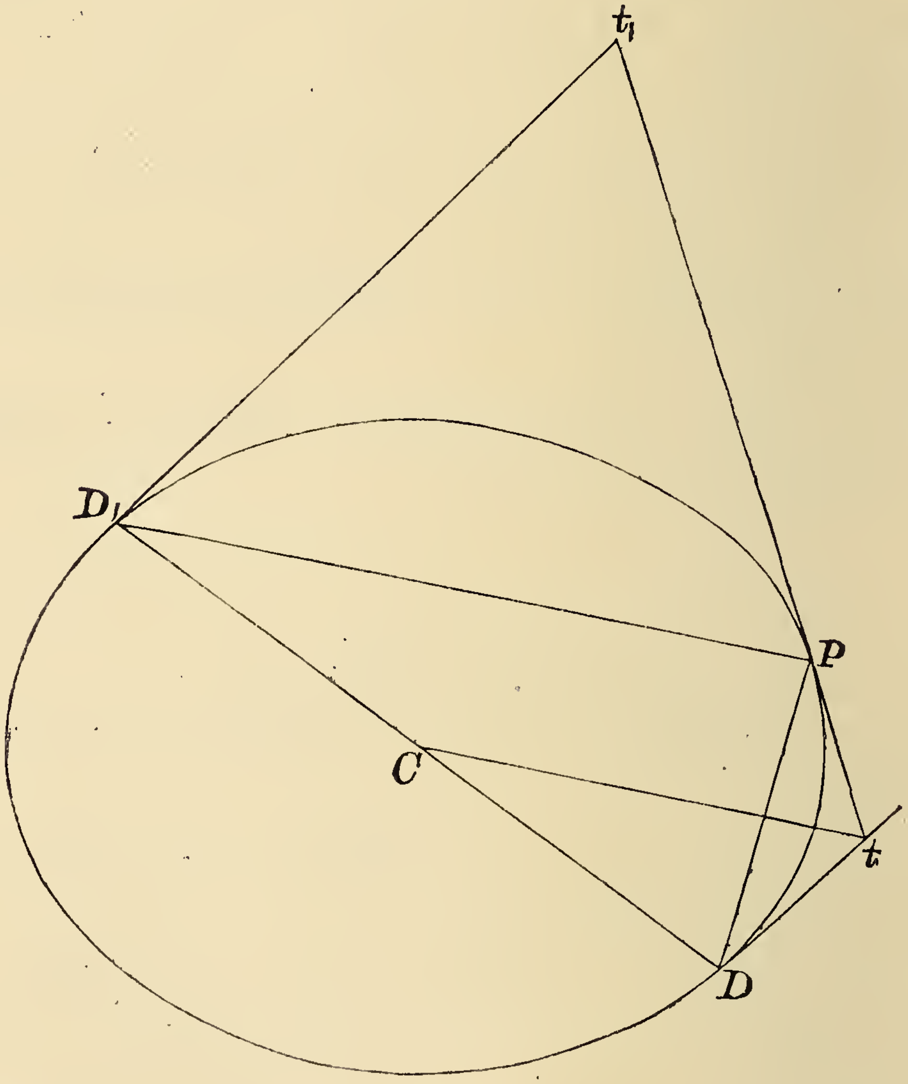
COR. Hence, the line joining the centre with the point of intersection of two tangents, bisects the chord joining the points of contact.

## PROP. XX.

*The points in which any two parallel tangents are intersected by a third tangent lie on conjugate diameters.*

Let $Dt$, $D_{,}t_{,}$ be two parallel tangents; $tPt_{,}$ a third tangent. $Ct$, $Ct_{,}$ are conjugate diameters.

Join $PD$, $PD_{,}$.



Then, because $Dt$, $D_{,}t_{,}$ are parallel tangents, [17. COR. 2. $DD_{,}$ is a diameter.

Therefore $PD$, $PD_{,}$ are supplemental chords.

But $Ct$, $Ct_{,}$ bisect $PD$, $PD_{,}$ respectively. [18. COR.

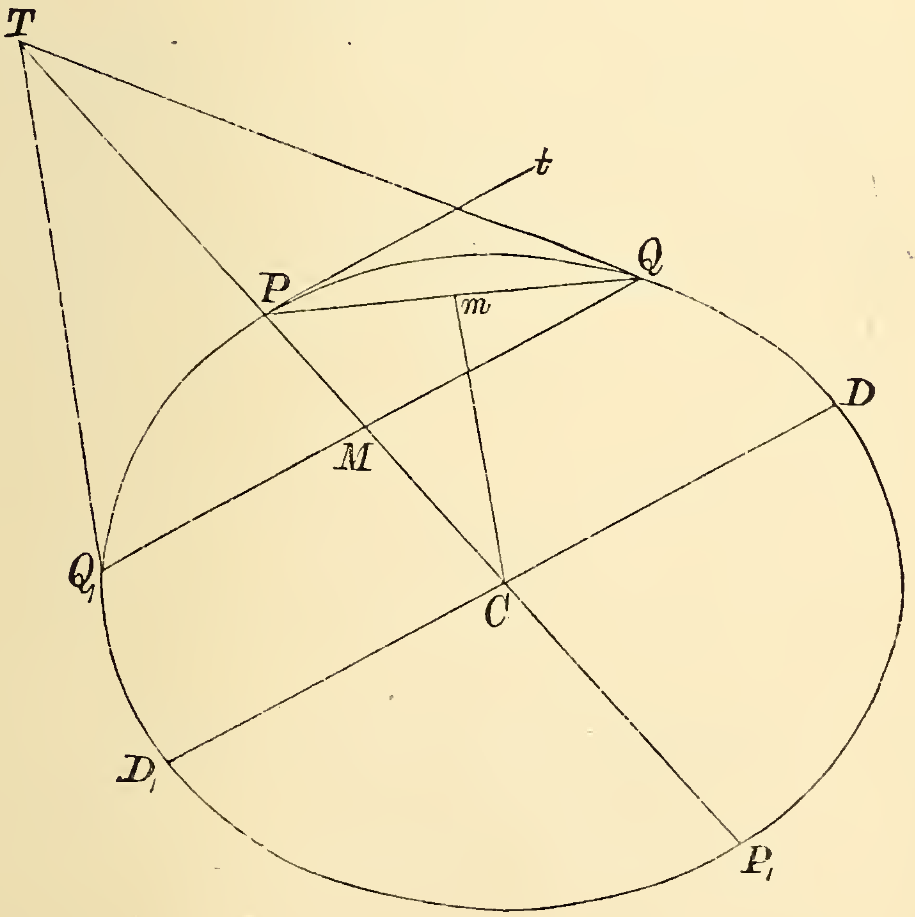
Therefore $Ct$, $Ct_{,}$ are conjugate diameters. Q. E. D. [16. COR. 2.

## PROP. XXI.

*If an ordinate be drawn to any diameter, the rectangle contained by the abscissæ : square of ordinate :: square of semidiameter : square of conjugate semidiameter.*

Let $QM$ be an ordinate to the diameter $PCP_{\prime}$, $CD$ the conjugate semidiameter;

$$PM \,.\, MP_{\prime} : QM^2 :: CP^2 : CD^2.$$



Produce $PM$ to meet the curve in $Q_{\prime}$.

Then $QM = MQ_{\prime}$.

But $PM \,.\, MP_{\prime} : QM \,.\, MQ_{\prime} :: CP^2 : CD^2$; [16.

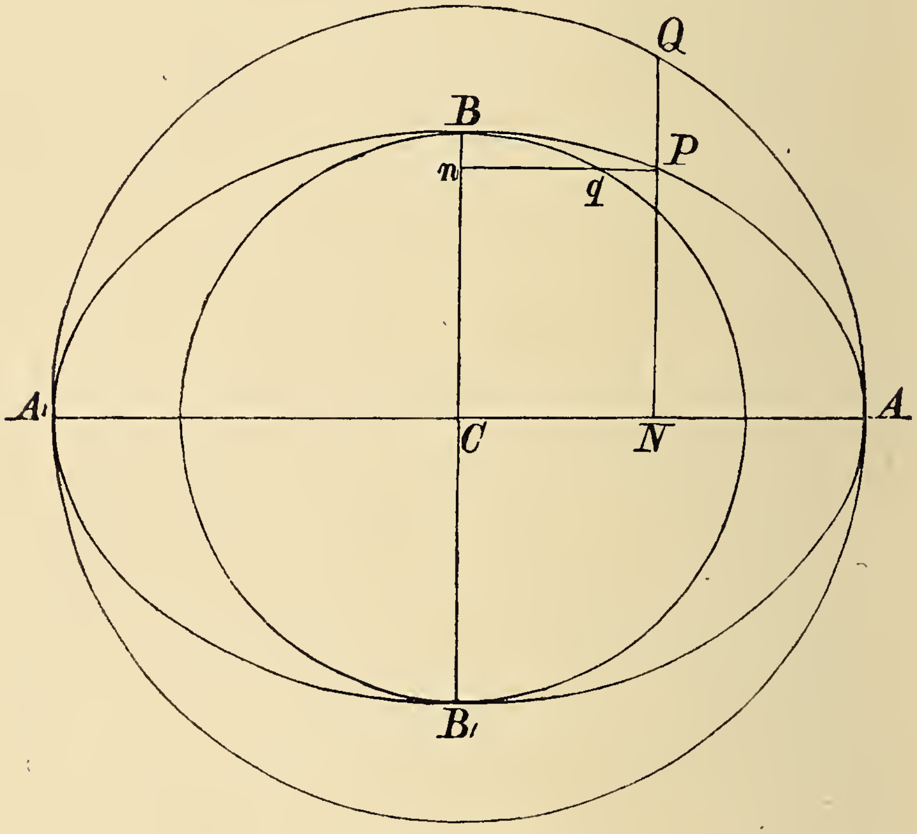
$\therefore PM \,.\, MP_{\prime} : QM^2 :: CP^2 : CD^2$.

Therefore if an ordinate &c. Q. E. D.

## PROP. XXII.

*If an ordinate PN to the major axis of the ellipse be produced to meet the auxiliary circle in Q,*

$$QN : PN :: CA : BC.$$



For $$AN.NA_{,} : PN^2 :: CA^2 : BC^2.$$ [21.

But $$AN.NA_{,} = QN^2;$$

$$\therefore QN^2 : PN^2 :: CA^2 : BC^2,$$

or $$QN : PN :: CA : BC.$$

COR. I. Exactly the same proof will shew that if $Pn$ the ordinate to the minor axis be produced to meet the circle on the minor axis in $q$,

$$qn : Pn :: BC : CA.$$

Cor. 2. Hence the area of the ellipse : area of auxiliary circle :: $BC : CA$. [L. 12.

Cor. 3. Also area of ellipse : area of circle on minor axis :: $CA : BC$. [L. 12.

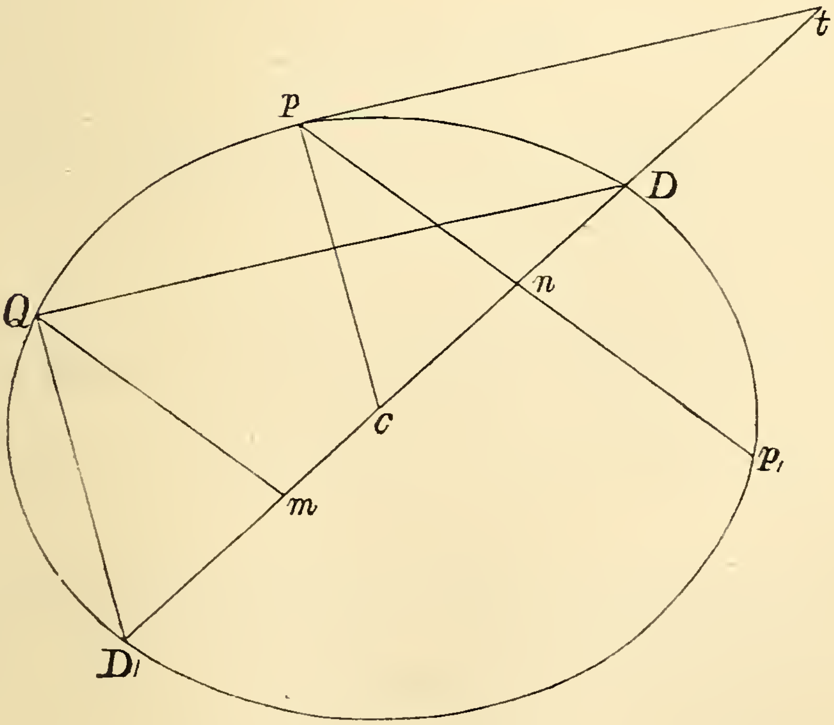
Cor. 4. The area of an ellipse is a mean proportional between the areas of circles described on the axes.

## Prop. XXIII.

*Any semidiameter is a mean proportional between the distances from the centre of the points where it meets the ordinate of any point, and the tangent at that point.*

If $CD$ is any semidiameter, $pn$, $pt$ ordinate and tangent at $p$,

$$Cn \,.\, Ct = CD^2.$$



Through $D$ draw $DQ$ the chord parallel to $pt$.

Join $D_{,}Q$, and draw $Qm$ ordinate to $CD$.

Then, because $DQ$, $D_{,}Q$, are supplemental chords, therefore they are parallel to conjugate diameters. [16.

But $DQ$ is parallel to $pt$.

Therefore $D_{,}Q$ is parallel to $Cp$. [17.

Also $Qm$ is parallel to $pn$.

Therefore the triangles $D_{,}QD$, $Cpt$ are similar and similarly divided.

Hence $Qm^2 : pn^2 :: Dm \,.\, mD_{,} : Cn \,.\, nt.$ [L. 1.

But $Qm^2 : pn^2 :: Dm \,.\, mD_{,} : Dn \,.\, nD_{,};$ [21.

$$\therefore Cn \,.\, nt = Dn \,.\, nD_{,}.$$

Add to each $Cn^2$;

$$\therefore Cn \,.\, Ct = Dn \,.\, nD_{,} + Cn^2$$

$$= CD^2. \quad \text{Q. E. D.}$$ [E. II. 5.

## DEFINITIONS.

24. Two points $nt$ situated on the same diameter $CD$, and on the same side of the centre, so that $Cn \,.\, Ct = CD^2$, are called *reciprocal points*.

25. The line through either of them, drawn parallel to the ordinates of $CD$, is called the *polar* of the other.

26. And conversely the other point is called the *pole* of this line.

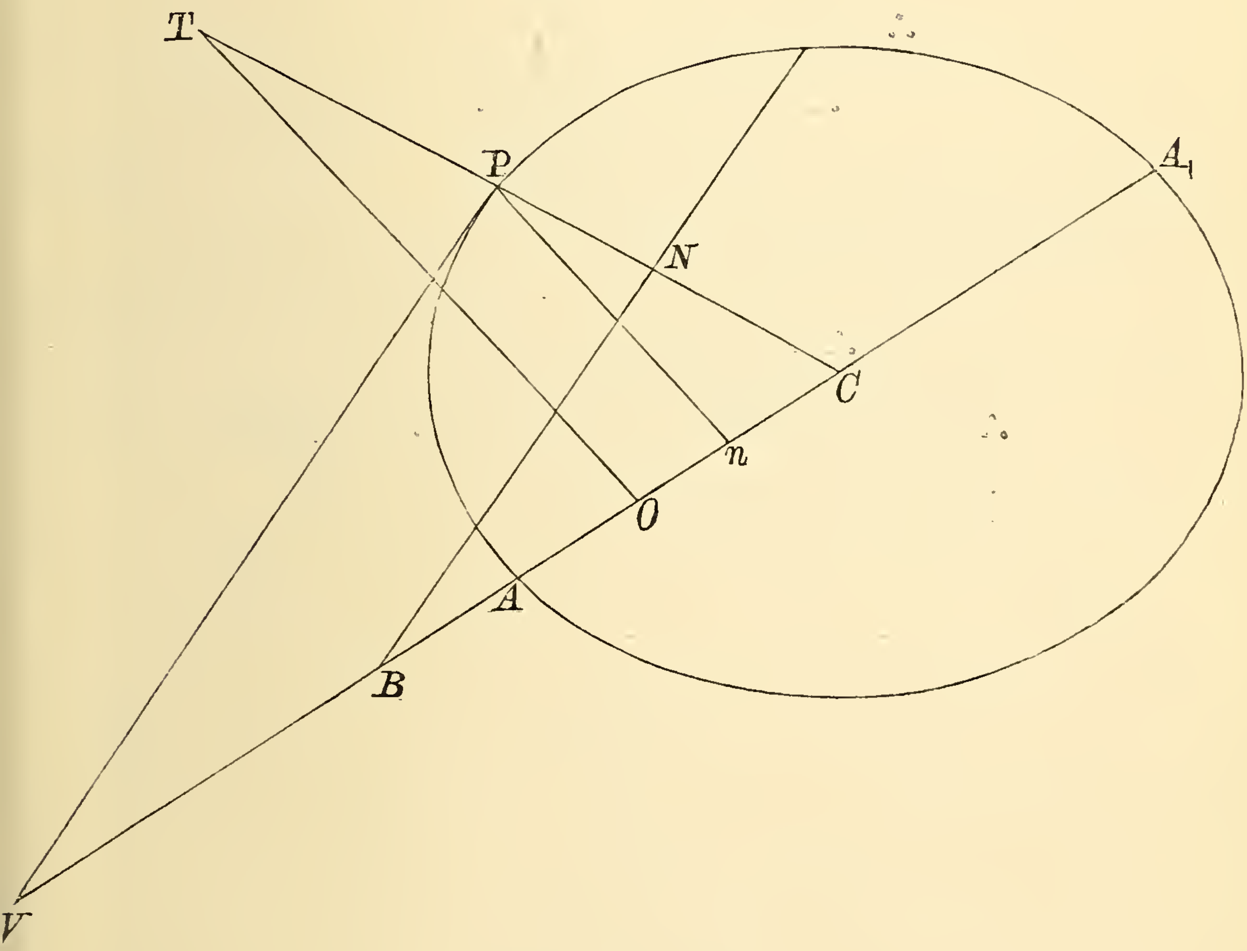
N.B. It is evident from the last proposition that the polar of an external point is the line joining the points of contact of tangents from that point.

And conversely the pole of any chord is the point of intersection of tangents at its extremities.

## PROP. XXIV.

*The polar of any point on a straight line passes through the pole of that line.*

Let *OT* be any straight line, meeting in *O* the diameter *CA*, to which it is an ordinate.



The polar of any point *T* on *OT* passes through *B* the pole of *OT*.

Join *CT* meeting the ellipse in *P*.

Through *B* draw *BN* an ordinate to *CP*, and draw *PV* the tangent at *P*, meeting *CA* in *V*, and parallel to *BN*. [18.

Also draw *Pn* an ordinate to *CA*, and therefore parallel to *OT*.

Then $Cn \, . \, CV = CA^2$ [23.

$= CO \, . \, CB$; [D. 24. 25.

$\therefore Cn : CO :: CB : CV$.

But $Cn : CO :: CP : CT$,

since $nP$ is parallel to $OT$;

and $CB : CV :: CN : CP$;

$\therefore CP : CT :: CN : CP$;

$\therefore CN \, . \, CT = CP^2$.

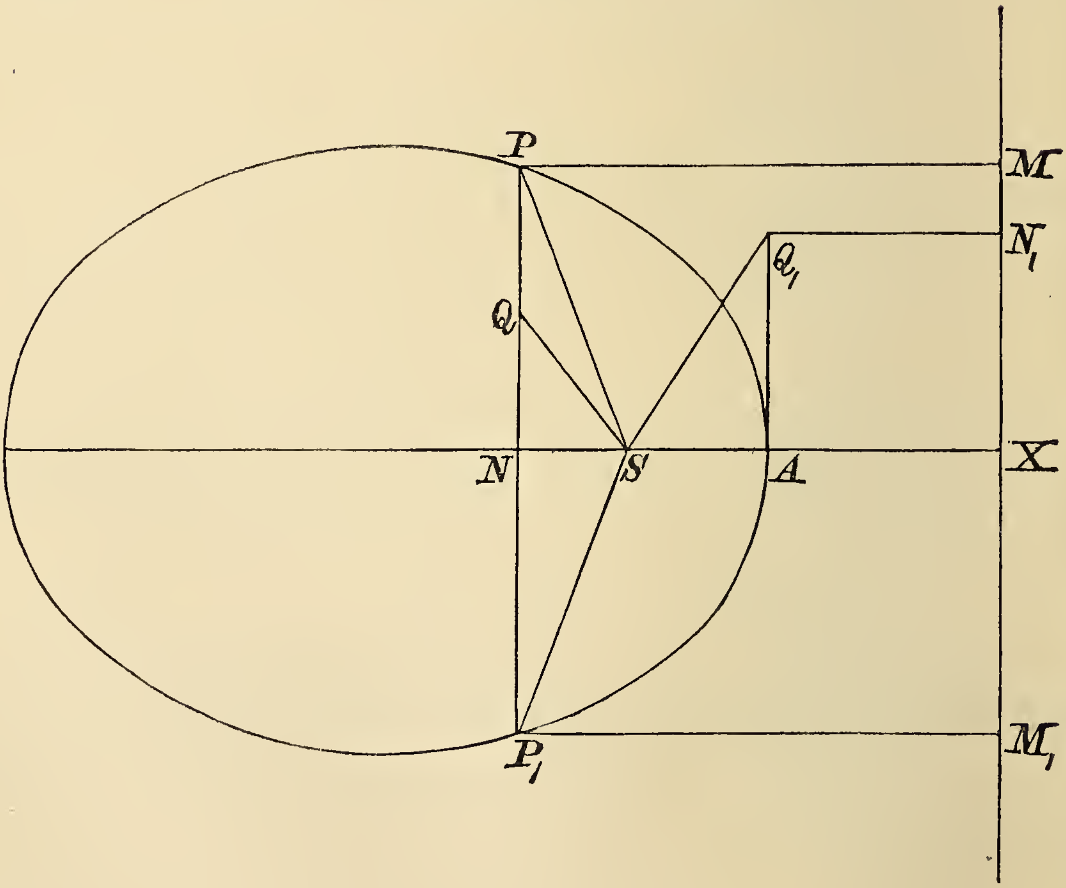
Hence $N_{,}T$ are reciprocal points. [D. 24.

But $BN$ is ordinate to $CP$;

$\therefore$ $T$ is the pole of $BN$; [D. 25.

$\therefore$ the pole of $BN$ lies on $OT$. Q. E. D.

COR. Let $X$ be the foot of the major axis.



Then $CS . CX = CA^2$; [6. Cor. 2.

$\therefore$ $S$ and $X$ are reciprocal points; [D. 24.

$\therefore$ $S$ is the pole of the directrix, and [D. 25.

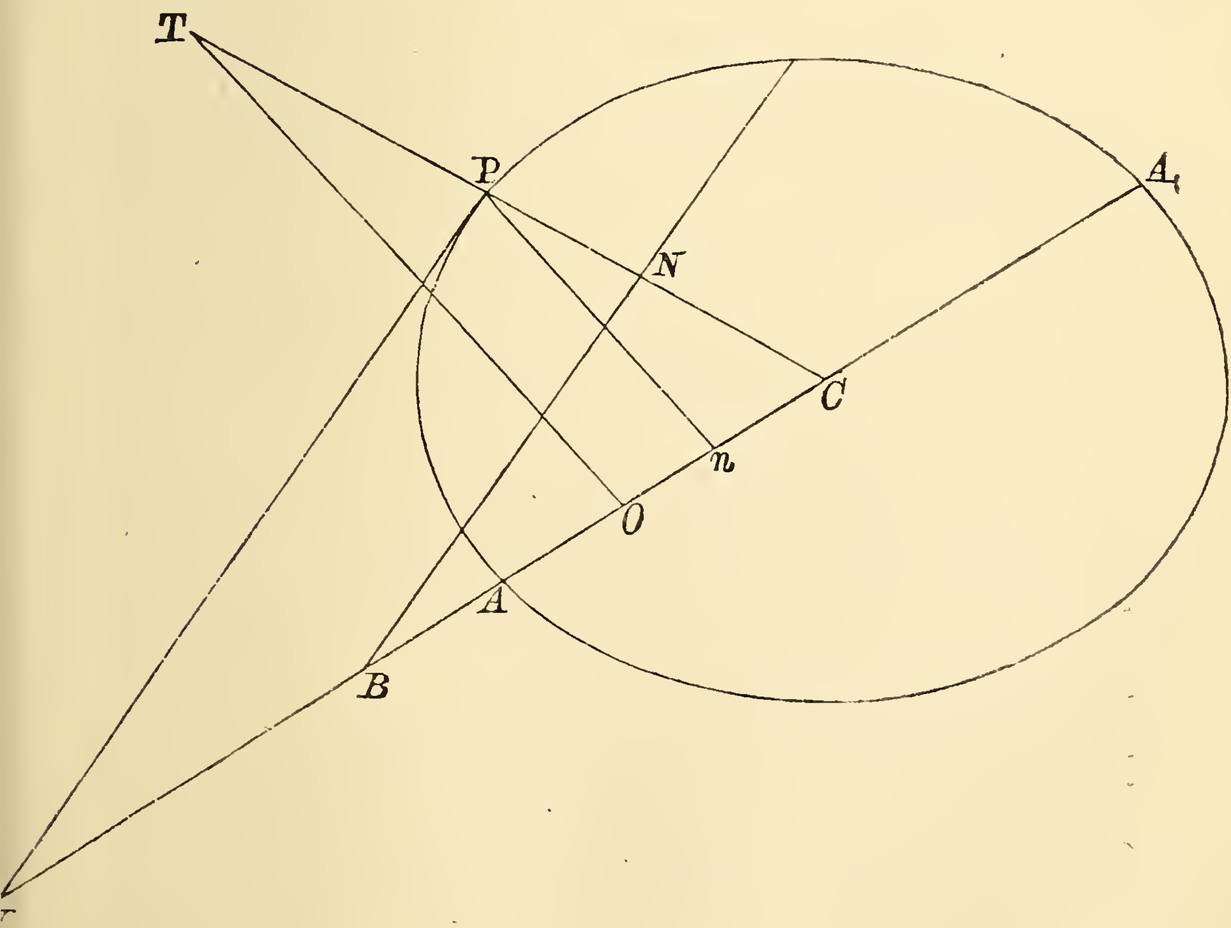
$X$ of the latus rectum. [D. 25.

Hence the pole of any focal chord lies on the directrix.

## Prop. XXV.

*The pole of a line which passes through a fixed point lies on the polar of that point.*

Let $OT$ be the polar of $B$; the pole of any line $BN$ through $B$ will lie on $OT$.



Draw $CPT$ the diameter which bisects chords parallel to $BN$.

Then, the construction of last proposition remaining,

$$Cn : CO :: CP : CT,$$

and $$CV : CB :: CP : CN;$$

$$\therefore Cn . CV : CB . CO :: CP^2 : CN . CT.$$

But $$Cn . CV = CA^2,$$ [22.

and $$CB . CO = CA^2;$$ [D. 24. 25.

$$\therefore CP^2 = CN . CT.$$

Hence $N$, $T$ are reciprocal points, [D. 24.

and $T$ is the pole of $BN$; [D. 25.

$\therefore$ the pole of $BN$ lies on $OT$. Q. E. D.

COR. The polar of any point on the directrix passes through the focus. [24 COR.

## PROP. XXVI.

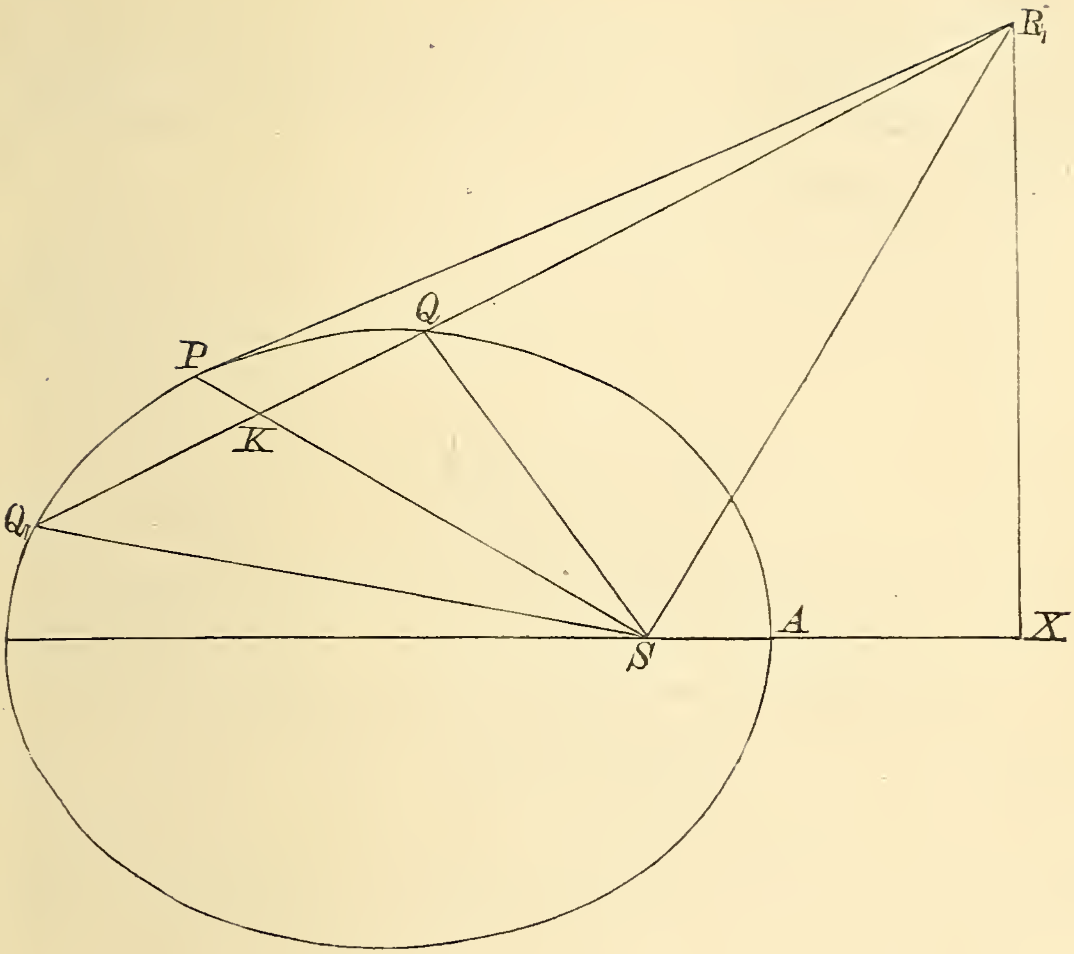
*The angle subtended at the focus by any chord is bisected by the straight line, through the focus, perpendicular to the line joining the focus to the foot of the chord.*

Let $R_{\prime}$ be the foot of a chord $QQ_{\prime}$.

Draw $SK$ bisecting $\angle QSQ_{\prime}$. Join $KR$.

Then $$QR : Q_{\prime}R :: QS : Q_{\prime}S;$$ [1.

$\therefore$ $SR$ bisects the angle exterior to $QSQ_{\prime}$. [E. VI. A.



But $SK$ bisects $QSQ_{,}$;

$\therefore\ KSR_{,}$ is a right angle;

or $KS$ is perpendicular to $SR$.

Therefore the angle, &c. Q. E. D.

## PROP. XXVII.

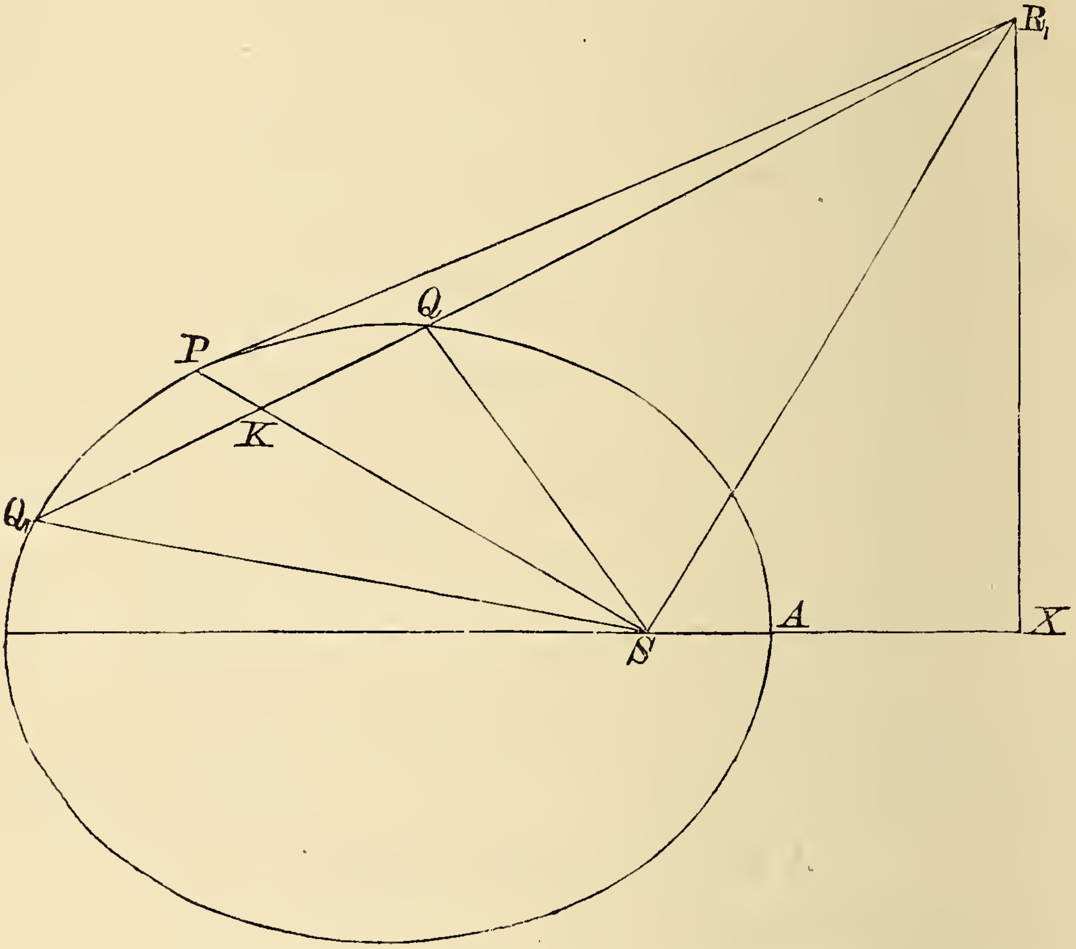
*Tangents from a point on the directrix subtend a right angle at the focus.*

In the last proposition let $Q_{,}$ move up to $Q$, and the angle $QSQ_{,}$ vanish.

Then $SQ$ will ultimately coincide with the bisector of the angle $QSQ_{,}$.

But $QQ_{\prime}$ becomes the tangent at $Q$, and the angle $QSR_{\prime}$ becomes ultimately a right angle; i. e. the tangent from $R_{\prime}$, a point on the directrix, subtends a right angle at the focus.

Therefore tangents, &c. Q. E. D.



Cor. 1. The line through the focus, perpendicular to $SR_{\prime}$, will be the polar of $R_{\prime}$.

For the polar of $R_{\prime}$ passes through $S$, [25. Cor.
and the polar of $R_{\prime}$ passes through $Q$; [D. 26. N.B.
hence $QS$ is polar of $R_{\prime}$.

But $QS$ is perpendicular to $SR_{\prime}$.

Cor. 2. Since the line $R_{\prime}S$ is perpendicular to $SQ$, it is evident that $R_{\prime}S$ is a tangent to the circle described with centre $Q$ through $S$.

Therefore $R_{,}S$ is parallel to a tangent drawn to the auxiliary circle from the foot of the diameter parallel to the tangent at $Q$. [12. Cor.

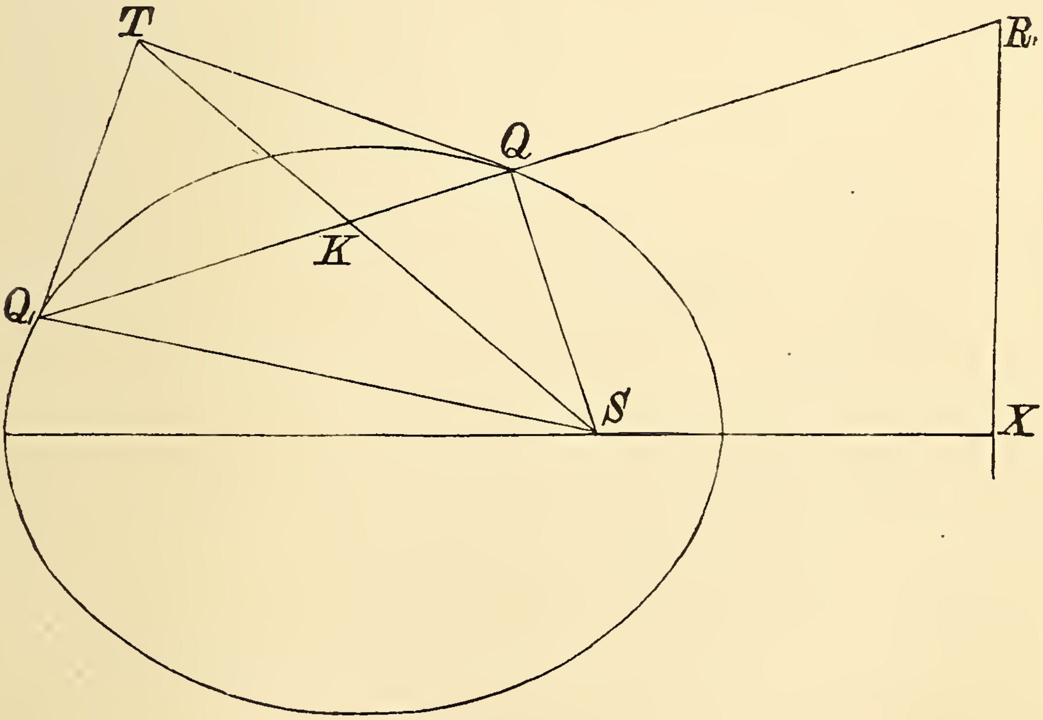
N.B. The line drawn from $S$ to the foot of the other tangent parallel to that at $Q$, will be parallel to the other tangent drawn to the auxiliary circle from the foot of the diameter.

## Prop. XXVIII.

*Tangents from an external point subtend equal angles at the focus.*

Let tangents at $Q$, $Q_{,}$, meet in $T$. The angles

$$QST = Q_{,}ST.$$



Let $R_{,}$ be the foot of $QQ_{,}$. Draw $SK$ bisecting the angle $QSQ_{,}$.

∴ $SK$ is perpendicular to $SR_{,}$; [26.

∴ $SK$ is the polar of $R_{,}$. [27, Cor. 1.

Then, since $T$ is the pole of $QQ_{\prime}R_{\prime}$; [D. 26. N.B.

the polar of $R_{\prime}$ passes through $T$. [24.

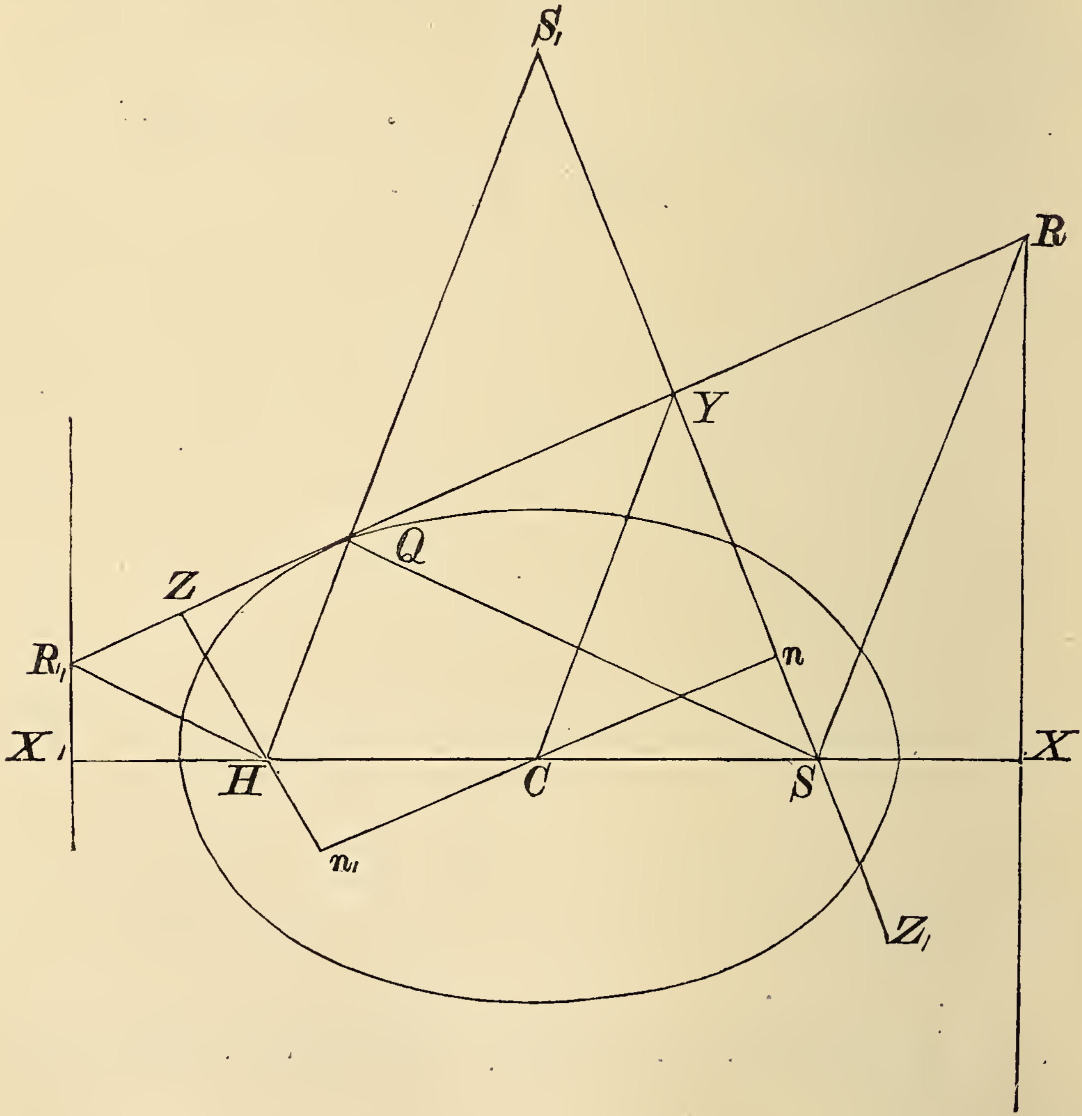
But $SK$ is the polar of $R_{\prime}$;

$\therefore$ $SK$ passes through $T$;

$\therefore$ $\angle QST = Q_{\prime}ST$. Q. E. D.

## Prop. XXIX.

*The focal distances of any point are equally inclined to the tangent at that point.*



Draw at $Q$ the tangent $RQR_{\prime}$ meeting the directrices in $RR_{\prime}$.

Then $SQ : HQ :: QR : QR_{,}$. [11. Cor.

And the angles $QSR \,.\, QHR_{,}$ are right angles. [26.

Therefore the triangles are similar; [L. 2.

and the angle $SQR = HQR_{,}$;

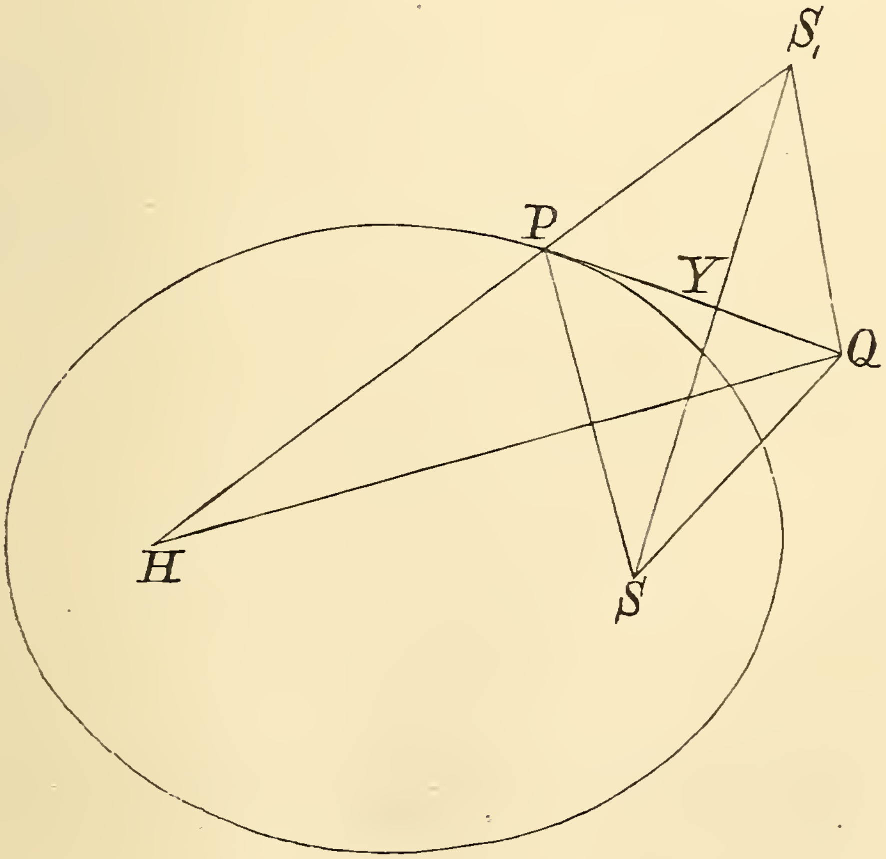
or $QR$ is equally inclined to the focal distances $SQ \,.\, HQ$. Q. E. D.

Cor. 1. Hence if $HQ$ be produced to $S_{,}$ so that $QS_{,} = SQ$, the straight line $SS_{,}$ will be bisected at right angles by the tangent.

For the tangent will bisect the vertical angle of the isosceles triangle $SQS_{,}$, and therefore bisect the base at right angles.

Cor. 2. Also $HS_{,} = HP + PS = AA_{,}$. [9.

Cor. 3. Every point on the tangent except the point of contact lies without the curve.



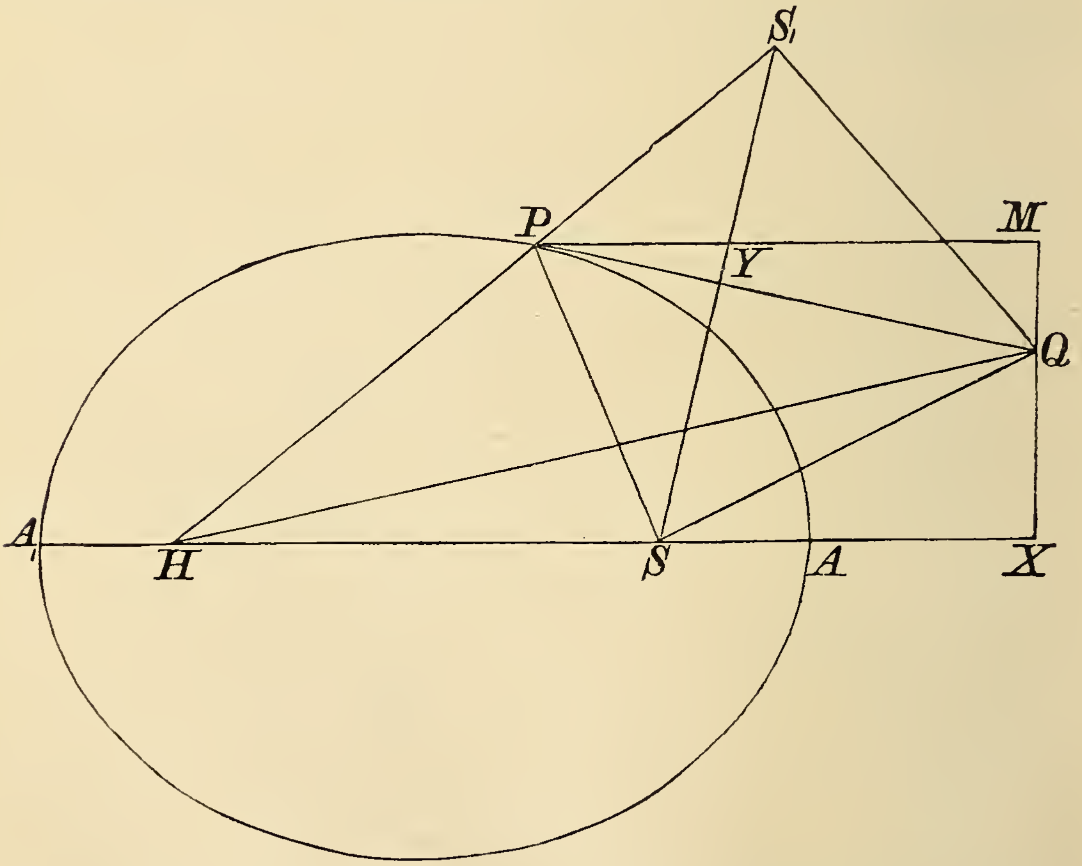
For let $Q_{\prime}$ be any other point on the tangent,
$P$ the point of contact.
Then because $QY$ bisects $SS_{\prime}$ at right angles;

$$Q_{\prime}S = Q_{\prime}S_{\prime};$$
$$\therefore\ SQ_{\prime} + HQ_{\prime} = S_{\prime}Q_{\prime} + H_{\prime}Q_{\prime};$$
$$\therefore \text{ is greater than } SS_{\prime} \text{ or } AA_{\prime};$$
$$\therefore\ Q_{\prime} \text{ lies without the curve.}$$ [10.

COR. 4. Since the normal is perpendicular to the tangent, and the tangent bisects the angle external to $SQH$, the normal will bisect the interior angle $SPH$.

N.B. The line $PG$ Prop. 9, is the normal to the ellipse at the point $P$.

N.B. If a tangent make an angle $\theta$ with the axis, and $\phi$ with the focal distance, $e \cos\theta = \cos\phi$.



Let $PQ$ be the tangent. Draw $PM$ perpendicular to the directrix.

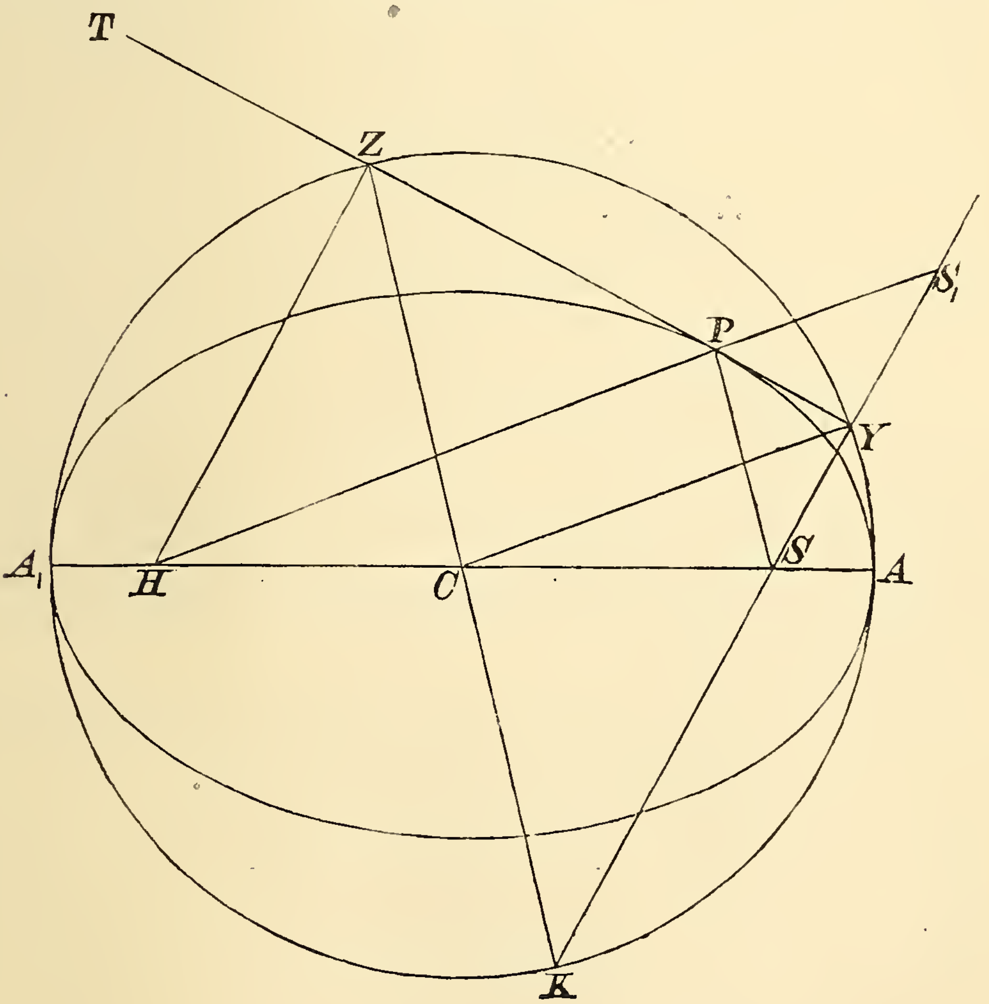
Then $PM = PQ \cos\theta$, since $QPM = \theta$;
but $PS = PQ \cos\phi$, since $SPQ = \phi$; [27.

$$\therefore PQ \cos\phi = e\,PQ \cos\theta,$$

$$\cos\phi = e \cos\theta. \quad \text{Q. E. D.}$$

## Prop. XXX.

*The rectangle contained by perpendiculars, let fall from the foci on any tangent, is equal to the square of the semi-axis minor; and the perpendiculars intersect the tangent on the circumference of the auxiliary circle.*



From $S$, $H$ draw $SY$, $HZ$ perpendicular to the tangent $YPZ$; $SY, HZ = BC^2$; and $Y$ and $Z$ lie on the auxiliary circle.

Join $CY$, and produce $HP$ to meet $SY$ in $S_{,}$.

Then $SS_{,}$ is bisected in $Y$. [27. Cor. 1.

But $HS$ is bisected in $C$;

$$\therefore \; CY = \tfrac{1}{2}SS_{,} = CA ; \qquad \text{[27. Cor. 2.}$$

$\therefore$ $Y$ lies on the auxiliary circle.

Produce $S_{,}Y$ to meet the circle again in $K$. Join $ZK$.

Then because $KYZ$ is a right angle;

$\therefore$ the circumference $ZAK$ is a semicircle. [E. III. 31.

Hence $KZ$ passes through $C$.

But $HC = CS$;

$$\therefore \; SK = HZ ;$$

$$\begin{aligned} \therefore \; SY \,.\, HZ &= KS \,.\, SY \\ &= AS \,.\, SA_{,} && \text{[E. III. 35.} \\ &= BC^2. && \text{[7.} \end{aligned}$$

Therefore the rectangle, &c. Q. E. D.

Cor. I. It is evident also that if any line be drawn through $S$ to the circumference of the auxiliary circle, the line perpendicular to it through the point of intersection will touch the ellipse.

For let $SY$ be such a line. Produce $SY$ to $S_{,}$ so that $SY = YS_{,}$, and join $S_{,}H$ meeting $YP$ in $P$.

Then as before $SS_{,} = 2CY = 2AC$.

But $S_{,}P = SP$;

$$\therefore \; HP + PS = AA_{,,}$$

or $P$ lies on the curve. [10.

Also $PY$ bisects the angle $SPS_{,}$;

$\therefore$ $PY$ is tangent at $P$. [29.

Cor. 2. Hence to draw a tangent to an ellipse from an external point $T$.

Upon $TS$ as diameter describe a semicircle cutting the auxiliary circle in $Y$;

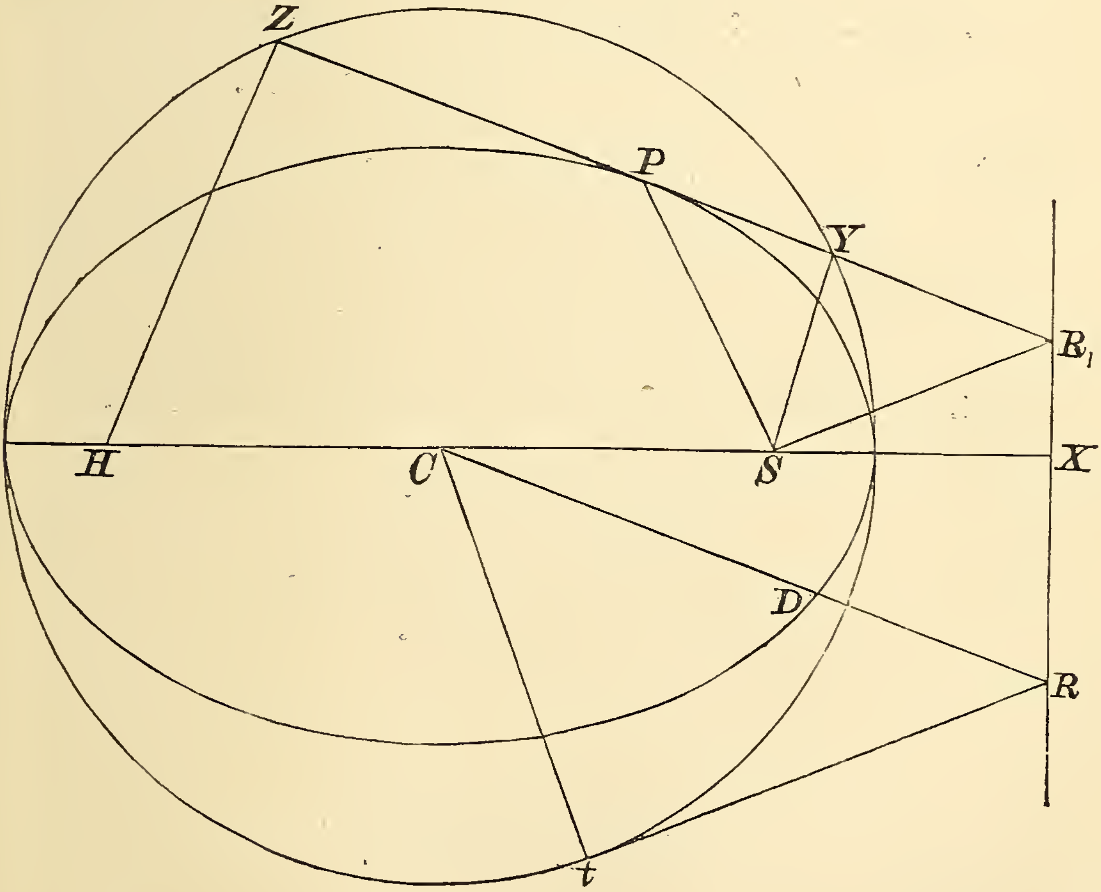
$\therefore$ $SYT$ is a right angle; [E. III. 31.

$\therefore$ $TY$ is the tangent from $T$.

Cor. 3. Hence, there can evidently be two tangents, and no more, drawn through an external point $T$.

### Prop. XXXI.

*The rectangle contained by the focal distances of any point is equal to the square of the semidiameter conjugate to the diameter through the point.*



If $CD$ is the semidiameter conjugate to $CP$,

$$SP \,.\, HP = CD^2.$$

Note. $HP$ should be joined in the figure.

Produce $PY$ to meet the directrix in $R_{,}$; and from $R$, the foot of the diameter parallel to $PY$, draw $Rt$ to touch the auxiliary circle.

Then $Rt$ is parallel to $SR_{,}$, [26. Cor. 2.
and $R_{,}SP$ is a right angle. [27.

Hence $$\angle SPY = \text{complement of } SR_{,}Y$$
$$= RCt;$$
$$\therefore\ SP : SY :: CR : Ct$$
$$:: CD : BC.$$ [14. Cor. 2.

But by similar triangles
$$HP : HZ :: SP : SY;$$
$$\therefore\ HP \,.\, SP : SY \,.\, HZ :: SP^2 : SY^2$$
$$:: CD^2 : BC^2.$$

But $$SY \,.\, HZ = BC^2;$$ [30.
$$\therefore\ HP \,.\, SP = CD^2.$$

Therefore the rectangle, &c. Q. E. D.

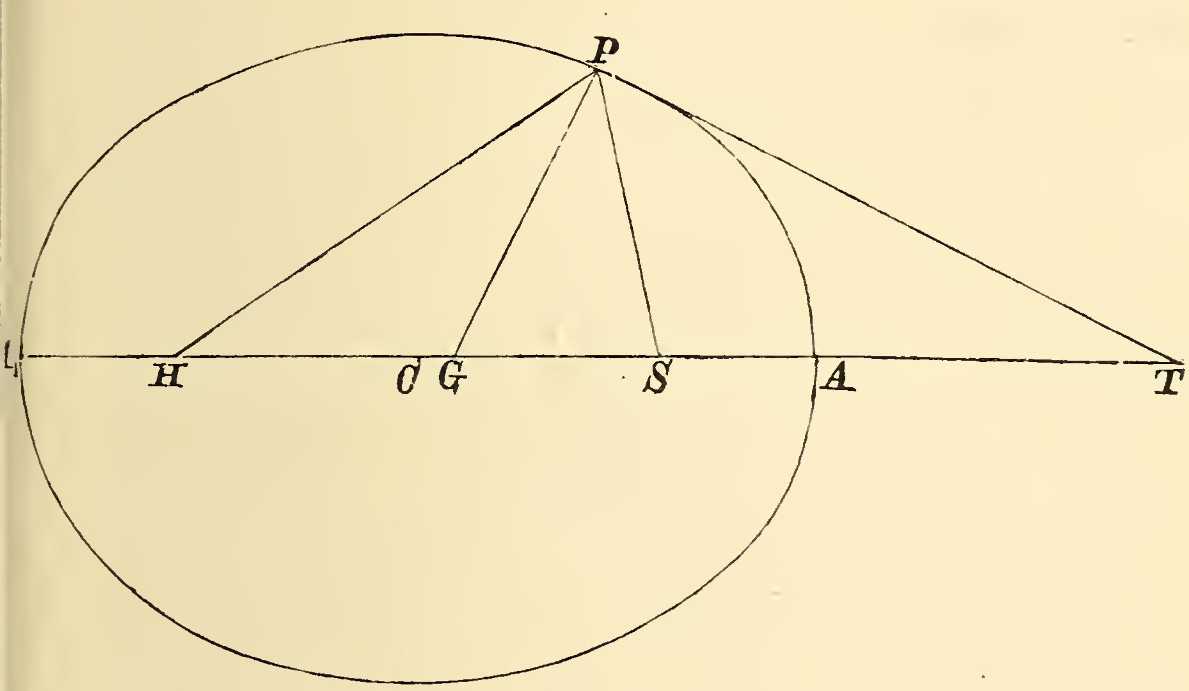
Cor. Hence if $PG$ meet the minor axis in $K$,
$$PG \,.\, PK = CD^2.$$

For $$PG \,.\, PK = SP \,.\, PH$$
$$= CD^2.$$

## Prop. XXXII.

*If the major axis is met by the normal and ordinate of any point in the points G, N,*
$$CG : CN :: e^2 : 1.$$

Because $PG$ is normal at $P$, $PG$ will bisect the interior angle $SPH$; [29. Cor. 4.



$\therefore SG : HG :: SP : HP.$ [E. vi. 3.

Again, because $PT$ bisects the exterior angle [29.
to $SPH$, $ST : HT :: SP : HP.$ [E. vi. a.

But $C$ is the middle point of $HS$;

$\therefore CG . CT = CS^2.$ [L. 8. Cor. 1.

But $CN . CT = CA^2$; [23.

$\therefore CG : CN :: CS^2 : CA^2$

$:: e^2 : 1.$ Q. E. D. [6.

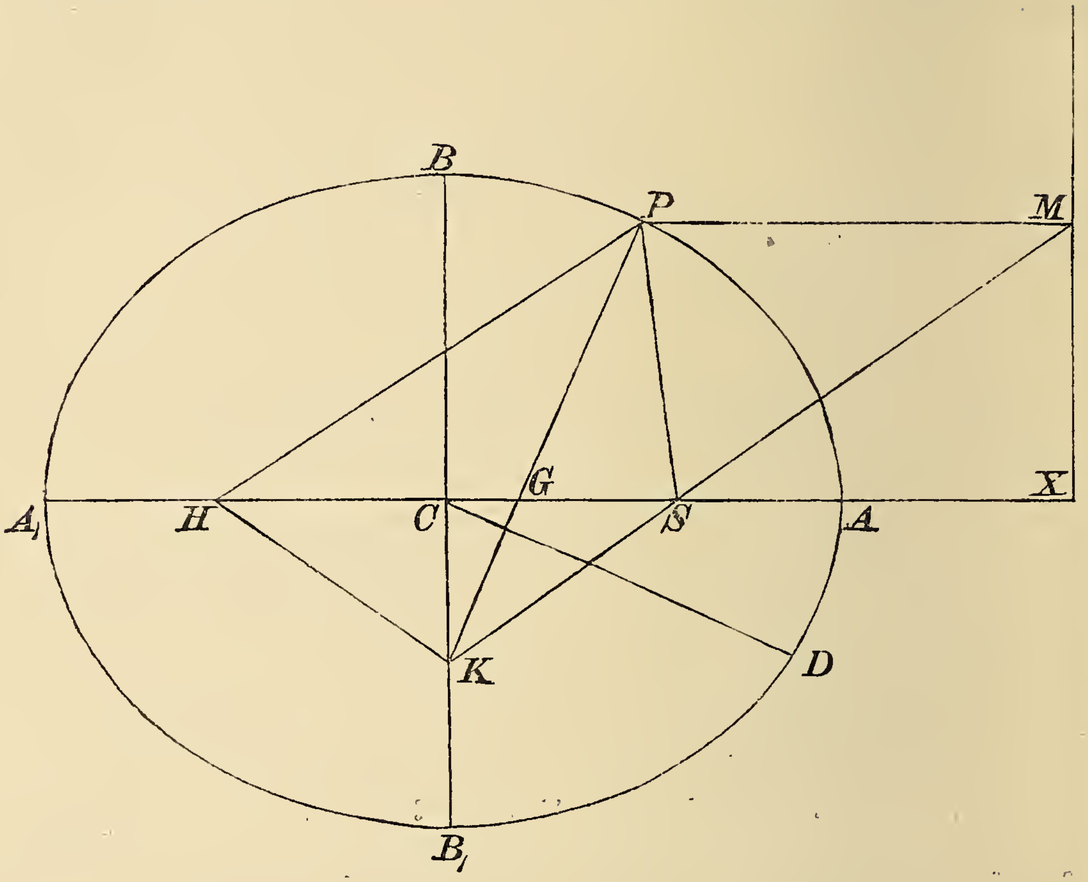
## Prop. XXXIII.

*If $G$ is the foot of the normal at $P$, $CD$ the conjugate diameter,*

$$PG : CD :: BC : CA.$$

Note. In the figure draw $PN$ perpendicular to $AA_1$.

Produce $PG$ to meet the minor axis in $K$.



Then $PG \,.\, PK = HP \,.\, SP$ [9. Cor. 4.

$= CD^2.$ [31.

But $PG : PK :: SM : KM$

$:: SX : CX$

$:: BC^2 : CA^2;$ [7. Cor. 3.

$\therefore PG^2 : PG \,.\, PK :: BC^2 : CA^2,$

$PG^2 : CD^2 :: BC^2 : CA^2,$

$PG : CD :: BC : CA.$ Q. E. D.

Cor. If the normal meet the minor axis in $K$,

$PK : CD :: CA : BC.$

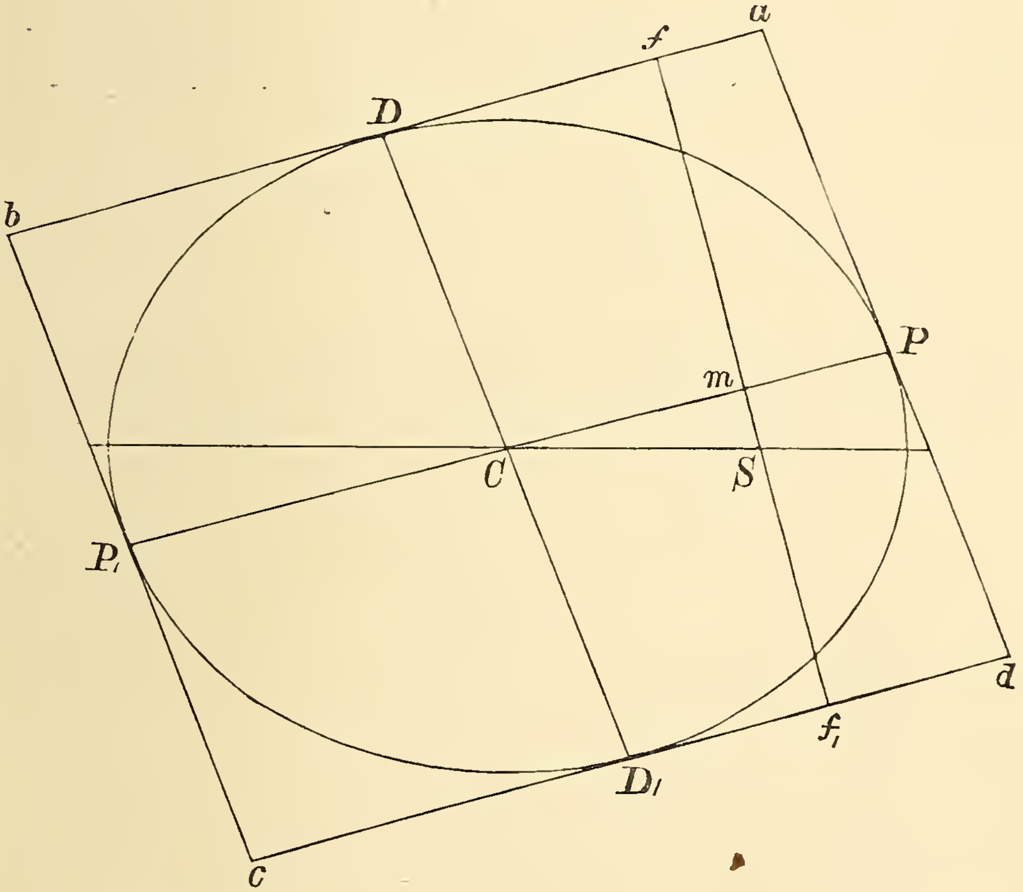
For $PK : CD :: CD : PG$

$:: CA : BC.$

## PROP. XXXIV.

*Parallelograms circumscribing ellipses at the extremities of conjugate diameters are of constant area, and equal to the rectangle contained by the axes.*

Let *abcd* be a parallelogram circumscribing the ellipse at the extremities of the conjugate diameters $PP_{,}$, $DD_{,}$.



Through $S$ draw $fSf_{,}$ perpendicular to $CP$ meeting in $f \cdot f_{,}$ the tangents $ab \cdot cd$ which are parallel to $PP_{,}$.

Then $f \cdot f_{,}$ are points on the auxiliary circle. [28.

Hence $PP_{,} \cdot ff_{,} = AA_{,} \cdot BB_{,}$. [14.

But $PP_{,} \cdot ff_{,} =$ parallelogram $bd$;

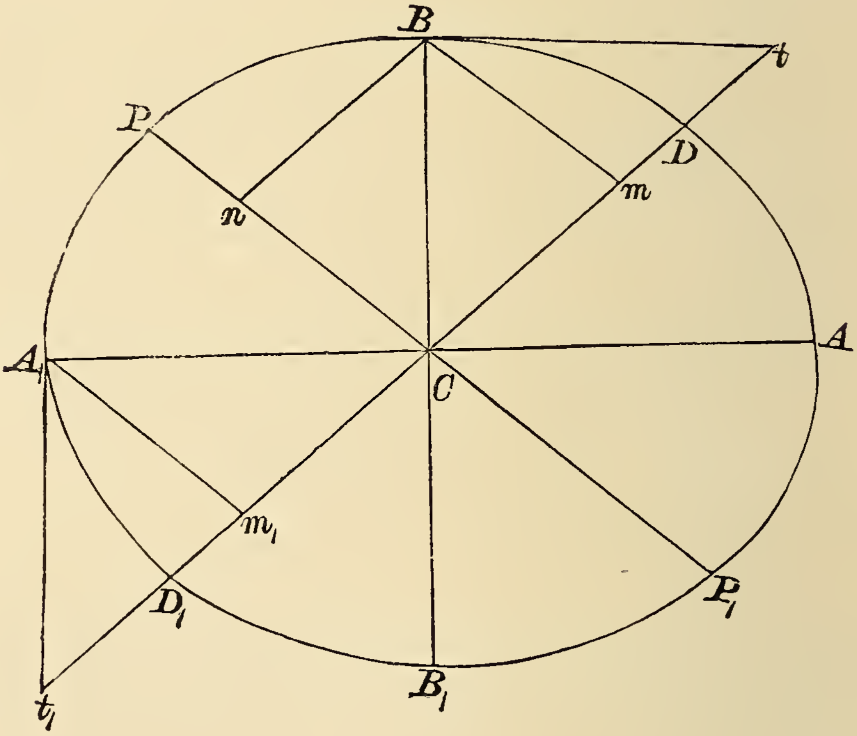
therefore the parallelogram *bd* is of constant area, and equal to the rectangle contained by the axes. Q. E. D.

## PROP. XXXV.

*If ordinates to any diameter be drawn from the vertices of two conjugate semidiameters, the sum of their squares will be equal to the square of the semidiameter to which they are parallel.*

Let $PCP_{,}$ be a diameter; $CD$ its conjugate; $A \,.\, B$ the vertices of two conjugate diameters; $Bn \,.\, An_{,}$ ordinates to $PP_{,}$;

$$An_{,}^{2} + Bn^{2} = CD^{2}.$$



Draw $Bt \,.\, At_{,}$ tangents at $B$ and $A$,

and $Bm \,.\, Am_{,}$ ordinates to $CD$.

Then $At_{,}$ is parallel to $CB$ and $At$ to $CA$. [18.

Hence the triangles $CBt$, $t_{,}AC$ are similar and similarly divided;

$$\therefore\ Cn_{,} : Ct_{,} :: mt : Ct, \qquad \text{[L. 1.}$$

$$Cm_{,}^2 : Cm_{,} . Ct_{,} :: Cm . mt : Cm . Ct.$$

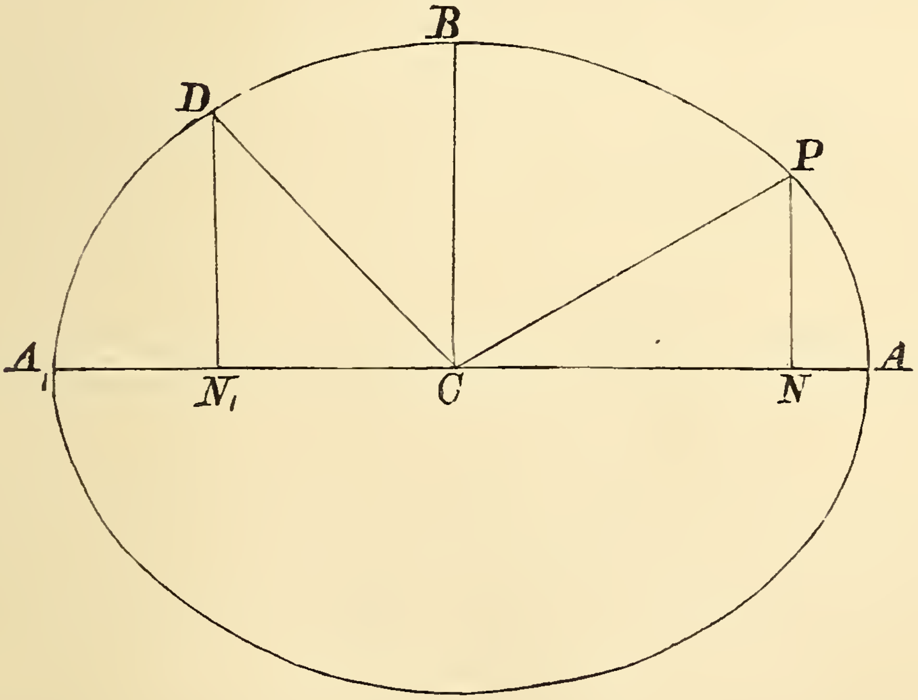
$$\text{But } Cm_{,} . Ct_{,} = CD^2 = Cm . Ct;$$

$$\therefore\ Cm_{,}^2 = Cm . mt;$$

$$\begin{aligned}\therefore\ Cm_{,}^2 + Cm^2 &= Cm . mt + Cm^2 \\ &= Cm . Ct \\ &= CD^2. \quad \text{Q. E. D.} \qquad \text{[23.}\end{aligned}$$

### Prop. XXXVI.

*The sum of the squares of two conjugate semidiameters is constant, and is equal to the sum of the squares of the axes.*



Let $CP . CD$ be two conjugate semidiameters,

$$CP^2 + CD^2 = CA^2 + CB^2.$$

Draw $PN \,.\, DN_{,}$ ordinates to the major axis, therefore $CN \,.\, CN_{,}$ will be equal to the ordinates to the minor axis.

$$\therefore\ PN^2 + DN_{,}^2 = CA^2, \qquad [35.$$

$$CN^2 + CN_{,}^2 = CB^2. \qquad [35.$$

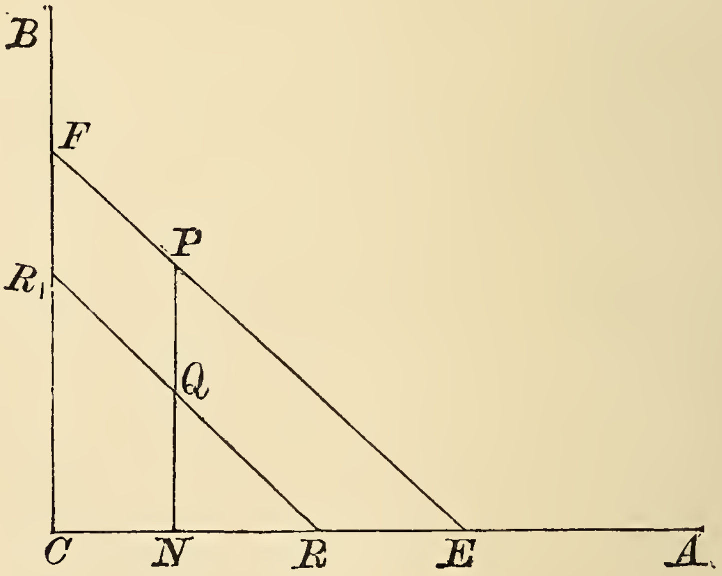
But $$PN^2 + CN^2 = CP^2, \qquad [\text{E. I. 47.}$$

and $$DN_{,}^2 + CN_{,}^2 = CD^2; \qquad [\text{E. I. 47.}$$

$$\therefore\ CP^2 + CD^2 = CA^2 + CB^2. \quad \text{Q. E. D.}$$

## PROP. XXXVII.

*If a straight rod slide between two perpendicular lines, so that its extremities may keep on the respective lines, the locus of any point of the rod will be an ellipse.*



Let $CA \,.\, CB$ be the lines, and let $EPF$ be a position of the rod.

From $P$ draw $PN$ perpendicular to $CA$.

In $PN$ take $NQ$ so that

$$QN : PN :: FP : EP.$$

Draw $RQR_{,}$ parallel to $FE$, meeting

$$CA,\ CB \text{ in } R,\ R_{,};$$

$$\therefore R_{,}Q = FP.$$

Also, $$QR : EP :: QN : PN;$$
$$:: FP : EP;$$
$$\therefore QR = FP.$$

Hence $RR_{,}$ is a line of length $2FP$,

and $P$ is its middle point;

$\therefore$ $BCA$ being a right angle,

$CQ = FP$, which is a fixed length;

and the ordinate $QN$ being produced in the ratio $FP : EP$, the locus of $P$ is an ellipse, whose axes are equal to $FP$ and $EP$. Q. E. D.

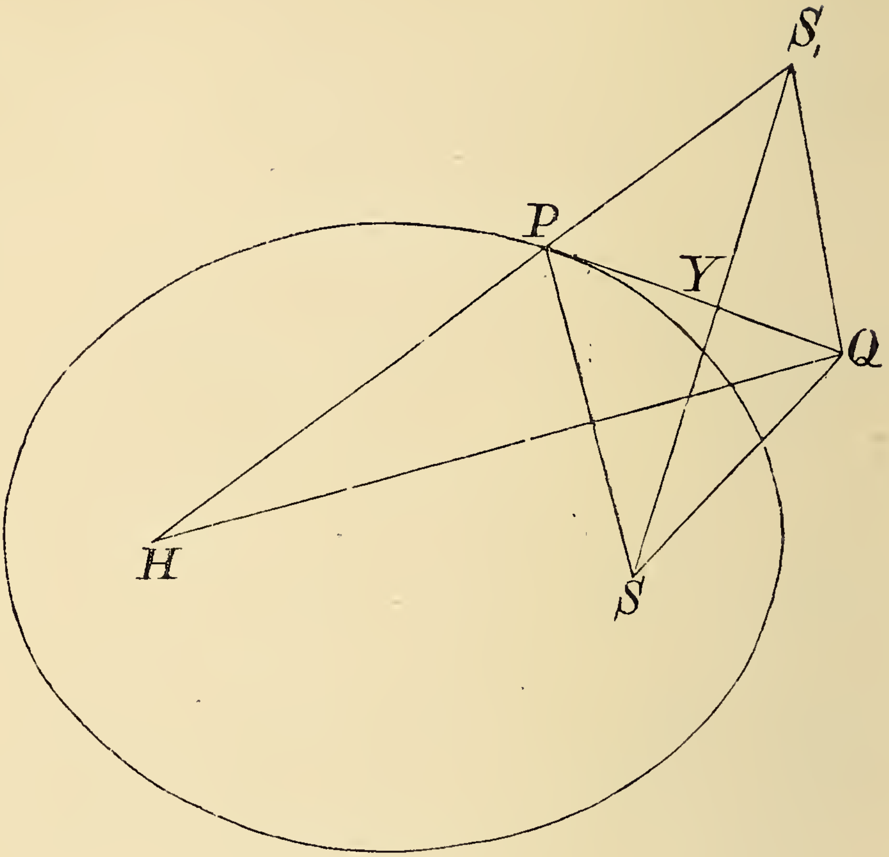
This is the principle employed in the elliptic compasses generally used by Cabinet Makers.

N.B. The following values of the cosine of an angle subtended at the focus by a tangent from an external point are often useful in analysis.

(1) If $r$ be the focal distance of the point; $r_{,}$ the distance from the other focus; $\alpha$ the angle subtended at the focus;

$$r_{,}^2 = r^2 + 4a^2 - 4ar \cos \alpha.$$

Let $H$ be the focus. $HQ = r$, $SQ = r_{,}$.



$$\text{Now } SQ = S_{,}Q = r_{,}, \text{ and } SS_{,} = 2a;$$

$$\text{and } \angle S_{,}HQ = \alpha;$$

$$\therefore \text{ by the triangle } S_{,}HQ,$$

$$r_{,}^2 = r^2 + 4a^2 - 4ar\cos\alpha.$$

(2) If $r$ be the focal distance of the point, $p$ the distance from the directrix, $ep = r\cos\alpha$.

Let $O$ be the point. $ON = p$, $SO = r$;

$POR$ the tangent; $R$ its foot; $P$ point of contact.

Then $RSP$ is a right angle.

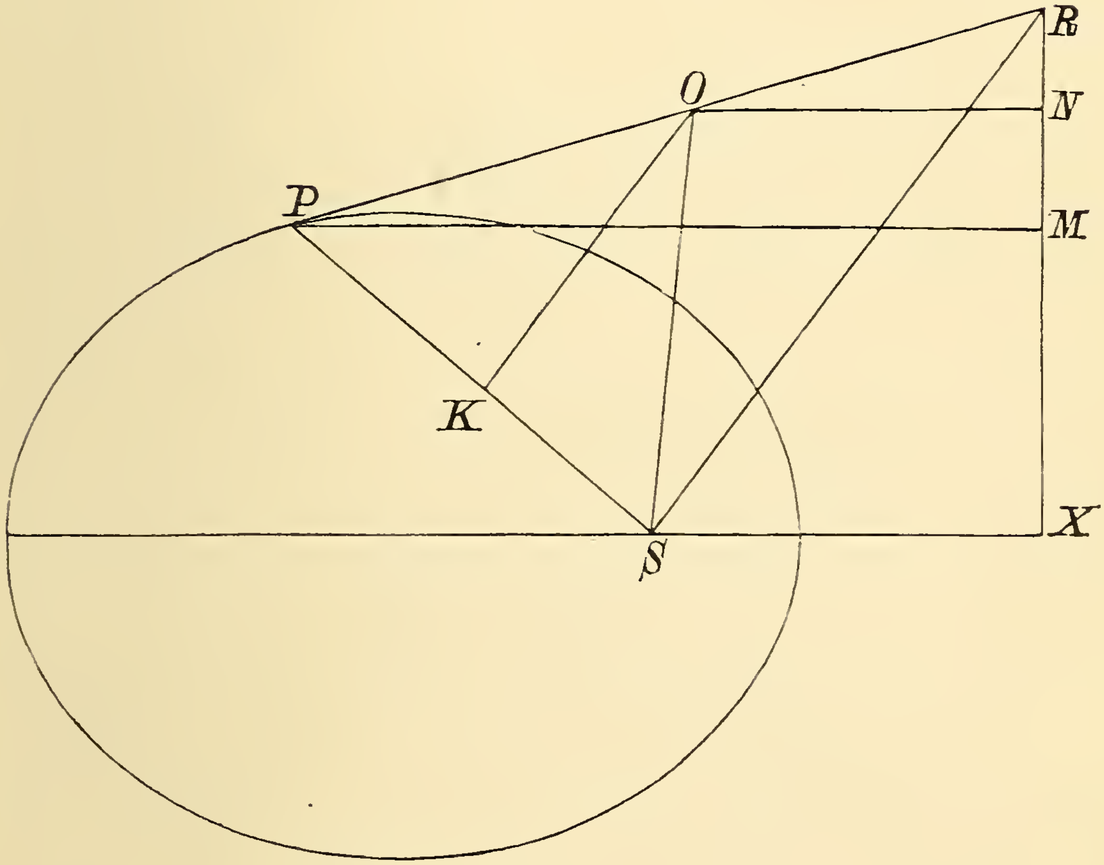
Draw $OK$ perpendicular to $SP$; $PM.ON$ perpendicular to the directrix.

Then $SK : SP :: OR : PR$

$:: ON : PM$;

$\therefore SK : ON :: SP : PM$

$:: e : 1$.



But $SK = r\cos\alpha$;

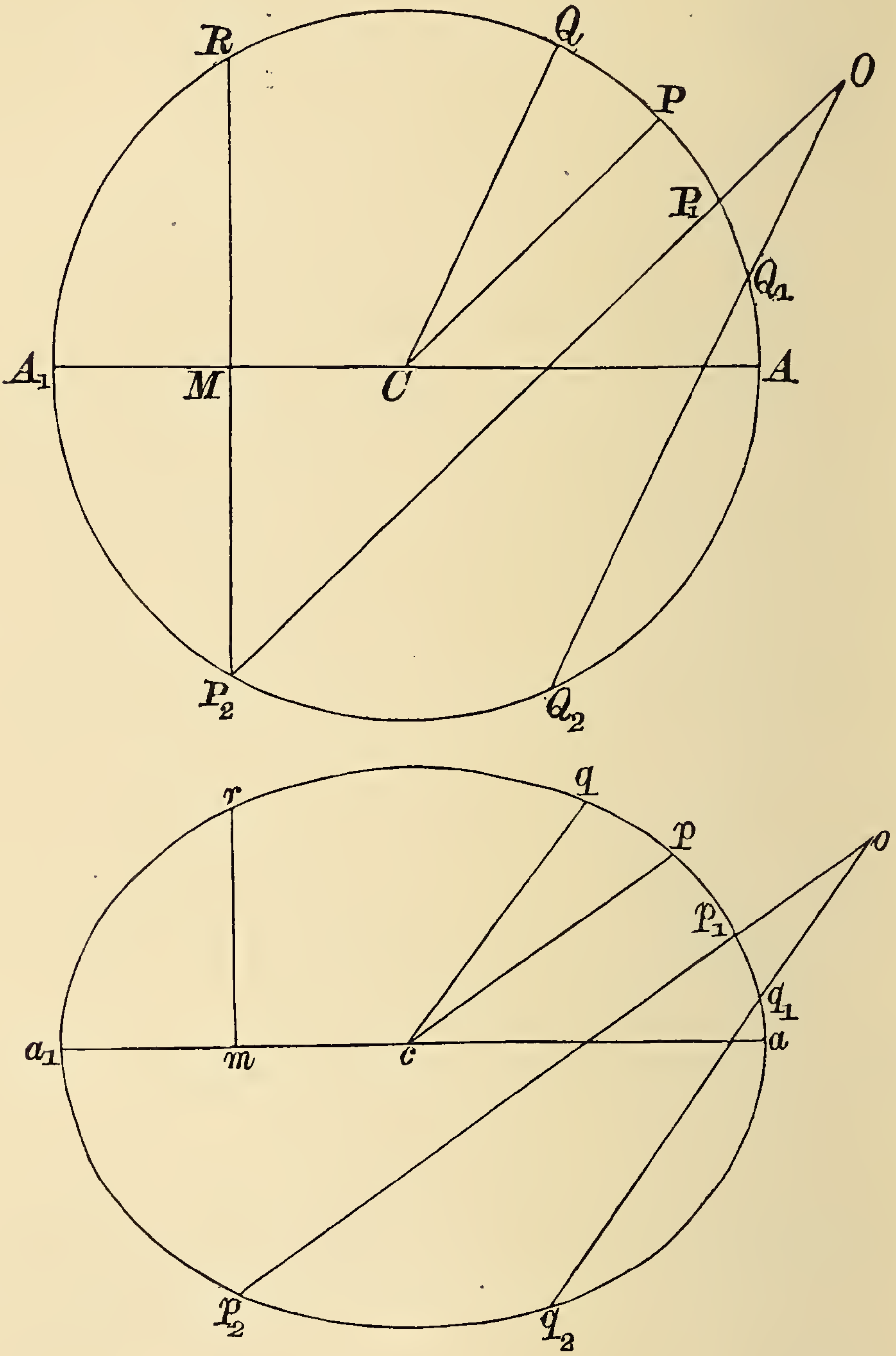
and $ON = p$;

$\therefore ep = r\cos\alpha$.

## Prop. XXXVIII.

*The orthogonal projection of a circle on a plane is an ellipse whose major axis is equal to the diameter of the circle.*

For let $AA_{,}$ be the diameter of the circle parallel to the intersection of the planes, $RM$ any ordinate of the circle,



$a$, $a_{,}$, $r$, $m$, the projections of these points.

Then $am = AM$, [L. 13. Cor. 2.

$a_{,}m = A_{,}M$, [L. 13. Cor. 2.

$rm = RM \cos \alpha$. [L. 13. Cor. 3.

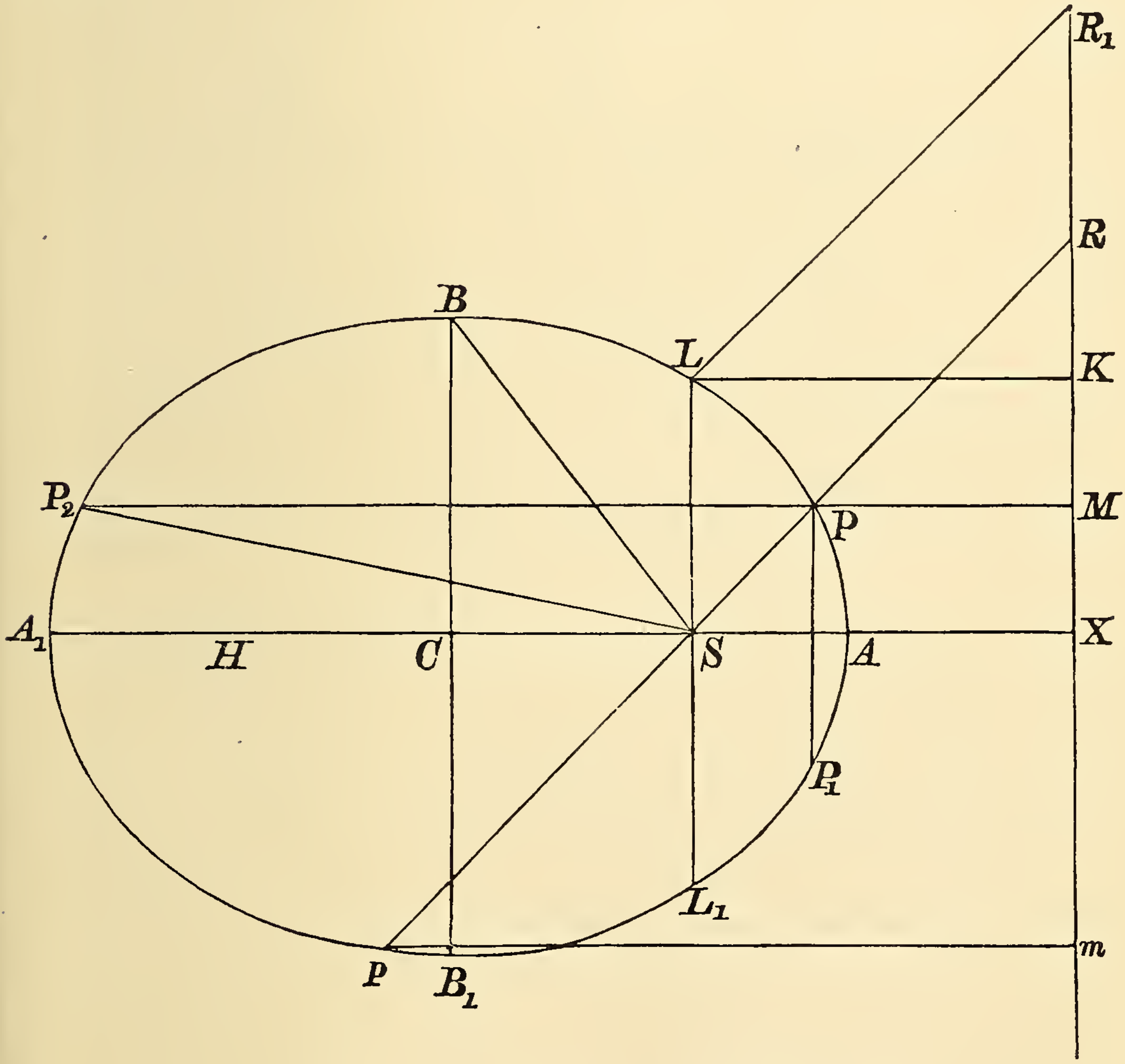
But $QM^2 = AM \cdot MA_{\prime}$; [E. III. 33.

$$\therefore\ PM^2 = am \cdot ma_{\prime} \cos^2 \alpha.$$

That is, the locus of $r$ is an ellipse whose major axis is equal to $aa_{\prime}$ or $AA_{\prime}$, and whose minor axis is equal to $AA_{\prime} \cos \alpha$. [21.

Hence the orthogonal projection, &c. Q. E. D.

COR. It is evident that the angle between the planes is $= SBC$.



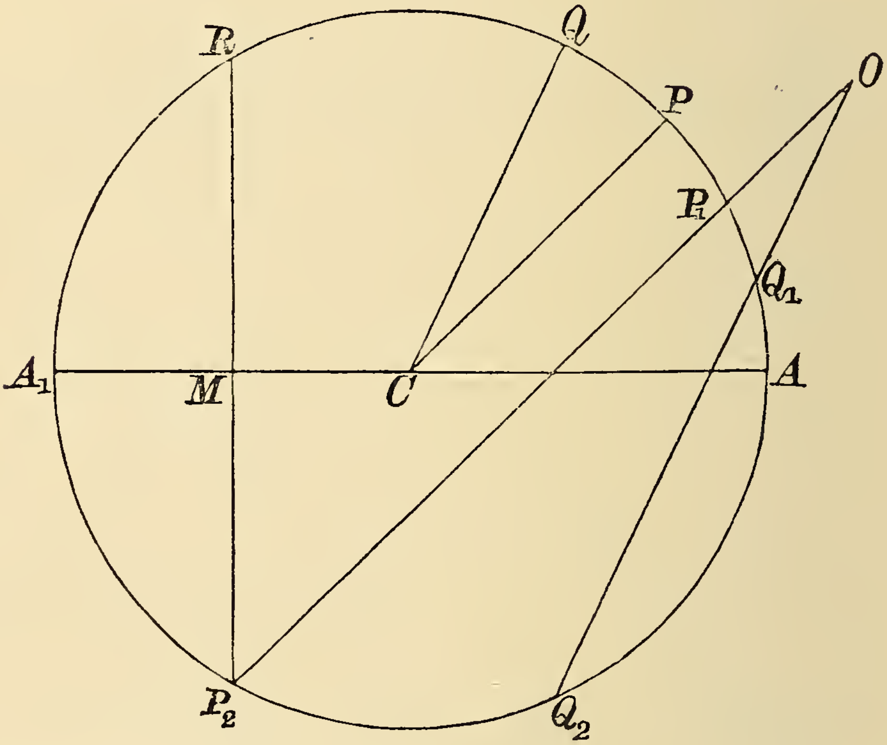
For $BC = SB \cos SBC$

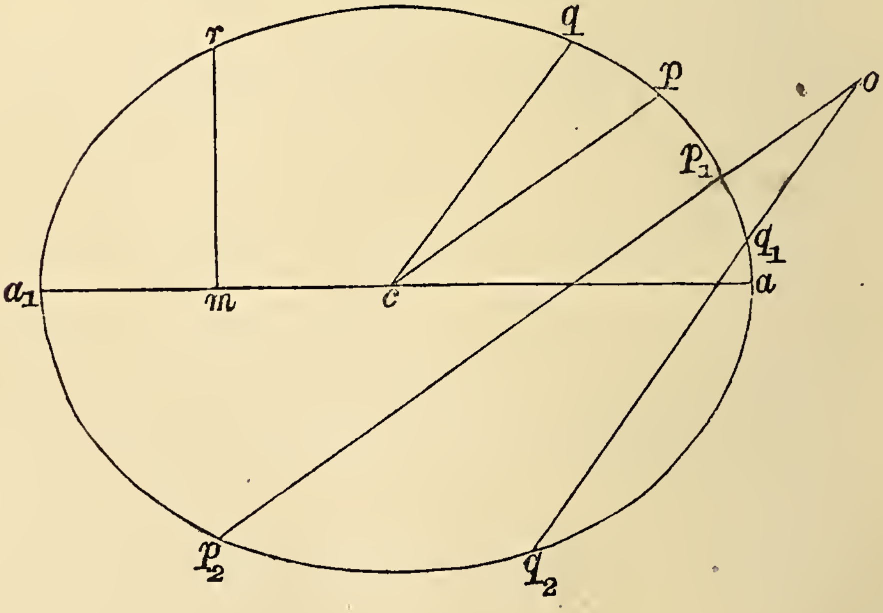
$= CA \cos SBC$; [6.

$\therefore\ \alpha = SBC.$

N.B. Many of the propositions already established may be readily proved by projections.

For example: Let $Op_1p_2 \,.\, Oq_1q_2$ be any two secants parallel to the semidiameters $cp \,.\, cq$.





Then $\quad Op_1 \,.\, Op_2 : Oq_1 \,.\, Oq_2 :: cp^2 : cq^2$.

For let $OP_1P_2 . OQ_2Q_2$ be the secants of the circle from which they are projected, $CP . CQ$ the parallel radii.

Then $CP^2 : OP_1 . OP_2 :: Cp^2 : op_1 . op_2$ [L. 13.

and $CQ^2 : OQ_1 . OQ_2 :: Cq^2 : oq_1 . oq_2$. [L. 13.

But $CP^2 = CQ^2$,

and $OP_1 . OP_2 = OQ_1 . OQ_2$; [L. III. 36.

$\therefore op_1 . op_2 : Oq_1 . Oq_2 :: cp^2 : cq^2$. Q. E. D.

Again, from the principles that a system of parallel chords of a circle are bisected by the diameter perpendicular to them; [E. III. 3.

And that the tangents at the extremities of this diameter are parallel to the chords; [E. III. 18.

All the properties of parallel chords of an ellipse established in Props. 13 and 18 may be deduced.

From this also it will readily be deduced that conjugate diameters are the projections of two diameters of a circle at right angles to one another, and from the fact that the angle in a semicircle is a right angle, [E. III. 31.

Prop. 17 follows immediately.

Evidently also parallelograms, circumscribing an ellipse at the extremities of conjugate diameters, will be the projections of squares circumscribing the circle; hence they are of constant area, as proved in Prop. 34, and also less than any other circumscribing parallelograms.

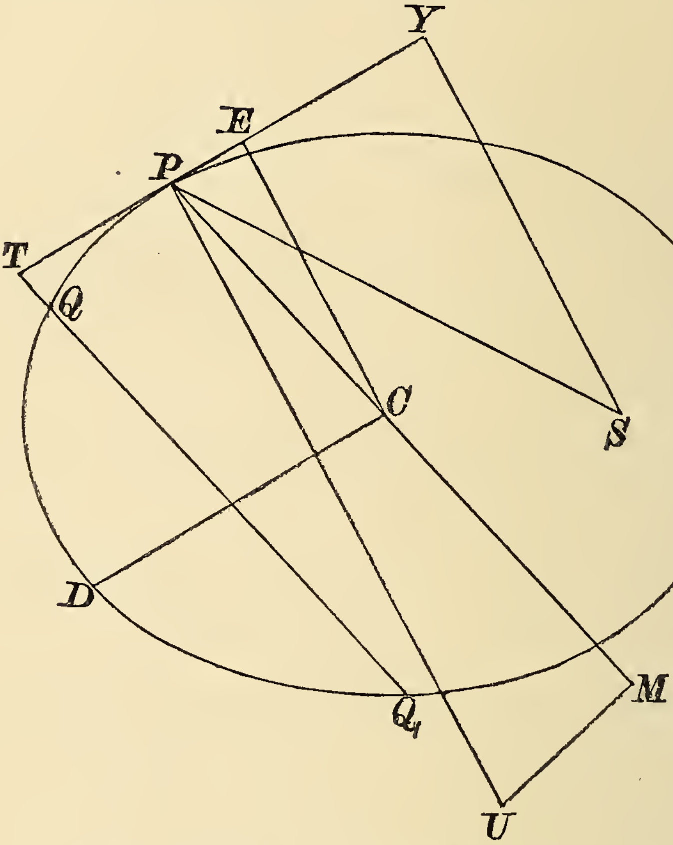
## PROP. XXXIX.

*The chord of curvature, at any point of an ellipse, through the centre, is a third proportional to the diameter through the point and its conjugate.*

Draw $PT$ tangent at $P$, and through $T$ draw $TQQ_{,}$, meeting the ellipse in $QQ_{,}$.

Also let $PM$ be the chord of curvature through $C$.

Draw $CD$ the conjugate semidiameter.



Then $CP^2 : CD^2 :: TQ \,.\, TQ_{,} : PT^2$ [18.

$:: \text{lt. } TQ : PM$ [L. 17.

$:: 2CP : PM,$

since the limit of $TQ$ when $T$ moves up to $P$ is $2CP$;

$$\therefore\ CP : CD :: CD : \tfrac{1}{2}PM,$$

or $PP_{,} : DD_{,} :: DD_{,} : PM$. Q. E. D.

## Prop. XL.

*The diameter of curvature* $= \dfrac{2CD^3}{CA \,.\, CB}$; *and the chord of curvature through the focus* $= \dfrac{2CD^2}{CA}$.

Let $PU$ be the diameter of curvature. From $C$ draw $CE$ perpendicular to $PT$.

Then $CE : CA :: CB : CD.$ [34.

Also $CE : CP :: PM : PU,$

by similar triangles;

$$\therefore CE \,.\, PU = CP \,.\, PM = 2CD^2; \quad [40.$$

$$\therefore PU : 2CD :: CD : CE$$

$$:: CD^2 : CD \,.\, CE$$

$$:: CD^2 : CA \,.\, CB;$$

$$\therefore PU = \frac{2CD^3}{CA \,.\, CB}.$$

Again, if $PK$ is the chord of curvature through $S$,

$$PU : PK :: SP : SY$$

$$:: CD : CB;$$

$$\therefore PK = \frac{2CD^2}{CA}. \quad \text{Q. E. D.}$$

## PROP. XLI.

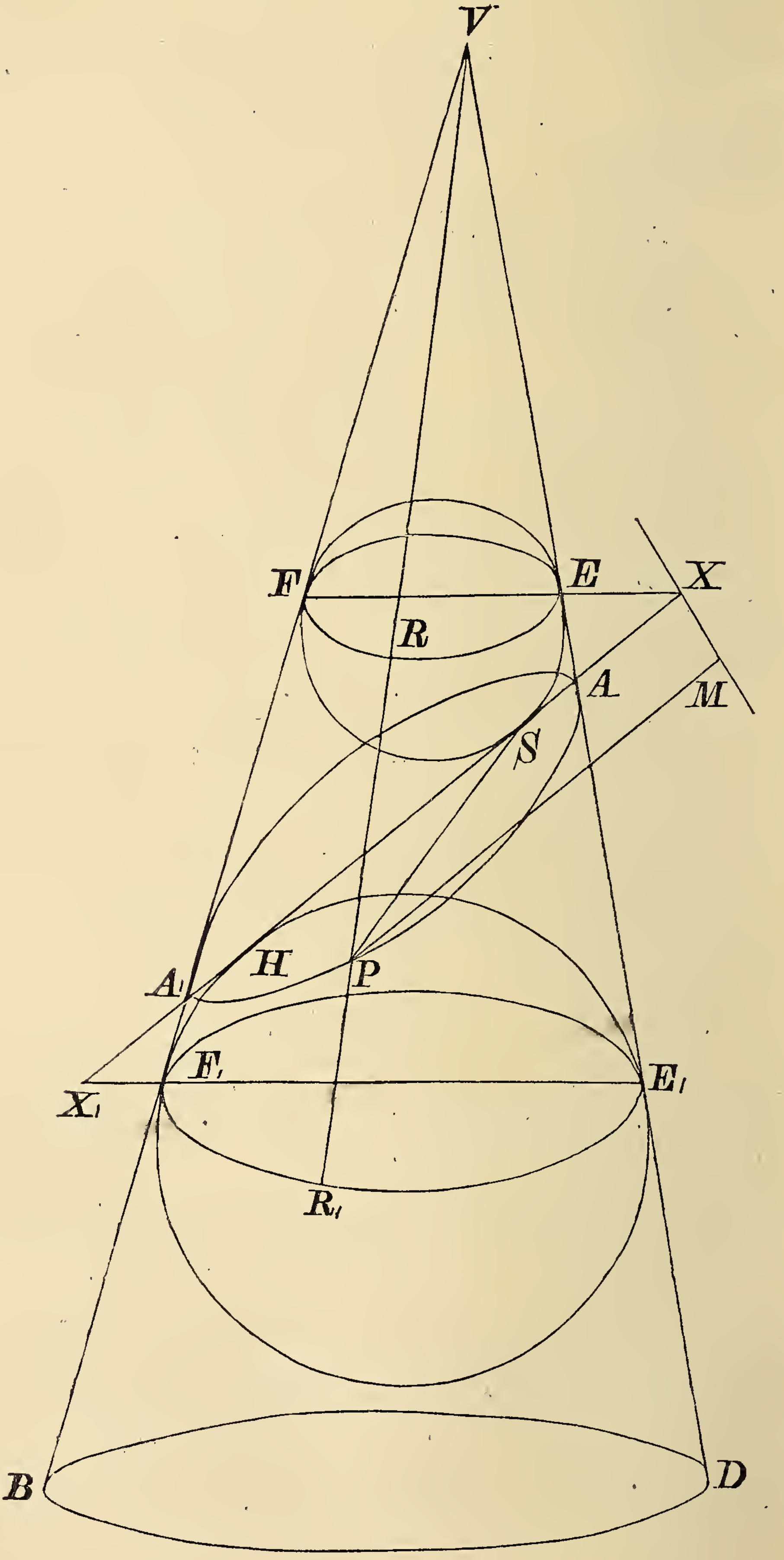
*The section of a right cone made by a plane cutting through the sheet is an ellipse.*

Let $VBD$ be a right circular cone, $APA_{\prime}$ a section of the cone by a plane cutting through the sheet.

Inscribe a sphere $EFS$, to touch the cone in the circle $ERF$, and also touch the plane of the section in $S$.

Similarly, escribe a sphere, touching the plane of the section in $H$, and the cone in $E_{\prime}R_{\prime}F_{\prime}$.

Through any point $P$ of the section draw through $V$ the line $VRPR_{\prime}$ to meet the circles $ERF$, $E_{\prime}R_{\prime}F_{\prime}$.



Join $SP.HP$.

Then $SP, PR$ are manifestly lines from $P$ which touch the sphere $ERF$;

$$\therefore SP = PR.$$

Similarly $$HP = PR,$$

$$\therefore SP + HP = RR_{\prime}.$$

Now the two circles $ERF$, $E_{\prime}R_{\prime}F_{\prime}$, being manifestly each perpendicular to the axis of the cone, are parallel to one another.

But $VRR_{\prime}$ makes a constant angle with the axis, therefore $RR_{\prime}$ is of a constant length.

Hence $$SP + HP = \text{constant}.$$

But $P$ is in a fixed plane through $S.H$;

$\therefore$ the section $APA_{\prime}$ is an ellipse of which $S$, $H$, are the foci. Q. E. D.

Cor. Also the directrix is the line of intersection of the planes $ERF$, $APA_{\prime}$.

For, let these planes intersect in $MX$.

Through the axis of the cone draw a plane perpendicular to $APA_{\prime}$.

This plane $VAME$ will obviously pass through the points of contact of the spheres $S$ and $H$.

Also being at right angles to each of the planes $ERF$, $APA_{\prime}$, it will be at right angles to their common section $XM$.

$\therefore AXM$ is a right angle.

Through $P$ draw $PM$, parallel to $AA_{\prime}$.

Hence $PM$ is drawn to the plane $ERF$ at a constant angle, for it is parallel to a fixed right line.

But $PR$ is also drawn to that plane at a constant angle, for it is a generating line of the cone.

$$\therefore\ PR : PM \text{ is a constant ratio.}$$

$$\text{But } PR = PS;$$

$$\therefore\ PS : PM \text{ is a constant ratio;}$$

$$\therefore\ XM \text{ is the directrix.}$$ Q. E. D.

## PROBLEMS.

1. If two fixed circles are touched, one externally and the other internally, by a variable circle, the locus of its centre is an ellipse.

2. If a circle cut an ellipse in four points, the chords of intersection are equally inclined to the axis.

3. If an ellipse have a given focus and touch two given lines, the locus of the other focus is a straight line.

4. Given the focus, length of major axis, and one point of an ellipse, the locus of the centre is a circle.

5. If a system of ellipses be described on the same diameter, and their diameter conjugate to this coincide in direction, the polar of any given point will pass through a fixed point.

6. If the axes of two ellipses are parallel, a circle can be described through their points of intersection.

7. If two ellipses, with parallel axes, touch one another, and intersect in two other points, the circle through their three points of meeting will touch both curves.

8. If $CP \,.\, CQ$ are two semidiameters at right angles to one another,

$$\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{CA^2} + \frac{1}{CB^2}.$$

9. If $pSp_{\prime}$, $qSq_{\prime}$ are two focal chords at right angles to one another,

$$\frac{1}{pp'}+\frac{1}{qq_{\prime}}=\frac{CA^2+CB^2}{2CA\,.\,CB^2}.$$

10. If $\phi$, $\phi_{\prime}$ be the angles $PSH$, $PHS$,

$$\tan\frac{\phi}{2}\,.\,\tan\frac{\phi_{\prime}}{2}=\frac{1-e}{1+e}.$$

11. If two intersecting ellipses have the same focus, their common chords pass through the intersection of the corresponding directrices.

12. If $PQ$ be the chord of the ellipse through $P$ parallel to the semidiameter $CK$, $PM$ the chord of curvature in the same direction, $CD$ conjugate to $CP$,

$$PQ : PM :: CK^2 : CD^2.$$

13. Deduce from Prop. 15 and 11 Cor. the fact that perpendiculars from the centre on a tangent vary inversely as the parallel diameters.

14. If $r_1$, $r_2$ are the semichords of curvature through the centre at the extremities of conjugate diameters, $a$ and $b$ the semiaxes,

$$r_1^{\frac{2}{3}}+r_2^{\frac{2}{3}}=\frac{a^2+b^2}{r_1^{\frac{2}{3}}r_2^{\frac{2}{3}}}.$$

15. If a circle be inscribed in the triangle $SPH$, the tangents to it from $P$ are equal to $SA$.

16. If two similar ellipses have their major axes in the same straight line, their common chord will bisect their common tangent.

17. Also if their major axes are equally inclined to the line joining the centres and towards the same parts.

18. If two chords of an ellipse $Pp$. $Qq$ meet on the directrix, prove that $PQ$, $pq$ subtend equal angles at the corresponding focus.

19. If tangents at $P$, $Q$ meet in $O$, the angles $SOP$, $HOQ$ are equal.

20. If $SP$. $HP$ meet in $EE_{,}$ the diameter conjugate to $CP$, shew that $PE = PE_{,} = CA$.

21. If $e = \sqrt{2}$, shew that $SBH$ is a right angle.

22. If $\theta$, $\theta_{,}$ are the inclination of two conjugate diameters to the major axis, shew that

$$\tan\theta \tan\theta_{,} = 1 - e^2.$$

23. Tangents at right angles to one another intersect in the circumference of a circle.

24. If an ellipse roll upon a similar and equal ellipse which is fixed in position, corresponding points being always in contact, the locus of either focus of the rolling curve is a circle.

25. Describe an ellipse, with given foci, to touch a given right line.

26. The tangent to an ellipse makes a greater angle with the focal distance than with the axis.

27. If an ellipse be drawn on a plane, shew how to find its axes by a geometrical construction.

28. If two conjugate diameters of an ellipse be given in position and magnitude, find the axes.

29. Find the excentricity of the ellipse in which the radius of curvature at the vertex of the minor axis is 8 times that at the extremity of the major axis.

30. Also if $SH = AX$.

31. If a chord $PQqp$ meet two concentric similar and similarly situated ellipses in $P$, $Q$, $q$, $p$ then

$$PQ = pq.$$

32. Also tangents to the interior curve cut off equal areas from the exterior.

33. The points of intersection of tangents at the extremities of conjugate diameters lie on a similar, concentric, and similarly situated ellipse.

34. The sum of two equal conjugate diameters is greater than the sum of any other pair of conjugate diameters.

35. Also the sum of the axes is less.

36. At the extremity of one of the equal conjugate diameters, the other extremity of the chord of curvature through the centre falls upon the ellipse; between this point and the extremity of the minor axis it falls without the ellipse; in the other quadrant within.

37. Given focus, minor axis, and tangent of an ellipse, the locus of the centre is a straight line.

38. Given focus, major axis, and tangent of an ellipse, the locus of the centre is a circle.

39. If a tangent to an ellipse meet two fixed tangents, the part intercepted will subtend a constant angle at the focus.

40. Two confocal ellipses cannot intersect.

41. If a tangent of an ellipse meet two parallel tangents, the rectangle contained by the parts into which the point of contact divides it, will be equal to the square of the parallel semidiameter.

42. The rectangles under the segments of parallel secants, drawn from points in a concentric, similar, and similarly situated ellipse, will be equal to one another.

43. If $Dt \,.\, D_{,}t_{,}$, the tangents at the vertices of a diameter $DD_{,}$, meet two conjugate diameters in $t \,.\, t_{,}$, and if $CP$ be conjugate to $DD_{,}$,

$$Dt \,.\, D_{,}t_{,} = CP^2.$$

44. If a triangle circumscribe an ellipse the lines joining the angles with the points of contact of the opposite sides pass through one point.

45. About a given ellipse describe a triangle of least area.

46. If in a given triangle the maximum ellipse be inscribed, the centre of the ellipse will coincide with the centre of gravity of the triangle.

47. The rectangle contained by the perpendiculars from any point on the curve to two fixed tangents varies as the square of the perpendicular on the chord of contact.

48. If from $G$ the foot of the normal at $P$, $GL$ be drawn perpendicular to $SP$, $PL = \frac{1}{2}$ latus rectum.

49. In Prop. 15 the angle subtended at $S$ by tangents to the ellipse from $O$ is equal to the angle subtended at $O$ by tangents to the circle from $S$.

50. The pole of a chord of an ellipse, which subtends at the focus a constant angle, less than the angle $BSC$, is an ellipse with the same focus and directrix.

51. The locus of the centres of circles inscribed in the triangle $HPS$ is an ellipse.

52. The latera recta of all ellipses cut from the same

right cone vary as the perpendiculars from the vertex of the cone on the plane of the section.

53. The chord of curvature through the focus of an ellipse is equal to the focal chord of the ellipse drawn parallel to the tangent.

54. If a system of chords of an ellipse pass through a fixed point, within or without the ellipse, the locus of their bisections is a similar ellipse.

55. If through an external point $O$ secants $Opp_{\prime}$, $Oqq_{\prime}$, &c. are drawn, and points $P$, $Q$, &c. taken on them so that $OP^2 = Op \cdot Op_{\prime}$, $OQ^2 = Oq \cdot Oq_{\prime}$; the locus of the points $P$, $Q$, &c. will be a similar ellipse.

56. The locus of the point whose distance from a given line bears to the tangent drawn from it to a given circle a given ratio, less than unity, is an ellipse.

57. A series of ellipses are described on a common diameter $DD_{\prime}$, the diameter conjugate to $DD_{\prime}$ being of a given length. If tangents be drawn from an external point $F$ in $DD_{\prime}$ to touch the ellipses, their points of contact will lie on a fixed circle.

58. If $Rr$ are the circumscribing and inscribed radii of the triangle $SPH$, $CD$ the semidiameter conjugate to $CP$, shew that $Rr$ varies as $CD^2$.

59. From any external point $O$ a secant $Opp_{\prime}$ is drawn to an ellipse. If in $pp_{\prime}$ a point $P$ be taken, so that $OP$ is the harmonic mean between $Op$, $Op_{\prime}$, shew that $P$ lies on the polar of $O$.

60. Hence shew that, if $Opp_{\prime}$, $Oqq_{\prime}$ are two secants, $pq \cdot p_{\prime}q_{\prime}$, and also $pq_{\prime}$, $p_{\prime}q$, intersect on the polar of $O$.

61. If four points on an ellipse be given, the polars of three fixed points are known.

62. Given two parallel chords of an ellipse, and the vertex of the bisecting diameter, find the centre.

63. Given two parallel chords of an ellipse and the centre (lying on the bisector of the two chords), find the vertices of the bisecting diameter and of its conjugate.

64. If $A$, $B$, $C$, $D$, $E$ are five points in an ellipse, shew how to find the point in which a line through $E$ parallel to $AB$ meets the curve again.

65. Describe an ellipse through five given points.

66. If two ellipses have a common focus and equal major axes, they cannot intersect in more points than two.

67. Draw a tangent to two ellipses with a common focus.

68. If $S$, $H$ are two points external to a closed curve, and $P$ be the point thereon for which $SP+PH$ is a minimum, shew that an ellipse described through $P$ with $S$ and $H$ as foci will touch the curve at $P$.

69. If $DQ$ a chord of an ellipse be produced to $F$, to meet in $T$ the tangent parallel to that at $D$, shew that $DQ \, . \, DT$ is equal to the square of the parallel diameter.

70. Describe an ellipse through three given points to touch a given line in a given point.

71. The rectangle under the distances of any variable point on an ellipse from two fixed tangents varies as the square of the distance from the point of contact.

72. Prove, by means of the focal distances, that the major axis is the longest straight line that can be placed in an ellipse.

73. If the focal distance of a point $Q$ on the ellipse be produced through $S$ so that $QS \, . \, SP$ shall be equal to $BC^2$, shew that the line through $P$ perpendicular to $SP$ will touch the auxiliary circle.

74. Of all pairs of conjugate diameters, the equal diameters are inclined to one another at the least angle. And this angle is $\sin^{-1} \dfrac{2}{\dfrac{a}{b} + \dfrac{b}{a}}$.

75. If straight lines be drawn from the focus inclined at a constant angle to the tangents, they will meet the tangents on the circumference of a given circle.

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